An introduction to the quark model

Jean-Marc Richard
Université de Lyon & Institut de Physique Nucléaire de Lyon
IN2P3-CNRS & UCB, 4 rue Enrico Fermi, 69622 Villeurbanne, France
j-m.richard@ipnl.in2p3.fr

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Abstract

This document contains a review on the quark model, prepared for lectures at the Niccolò Cabeo School at Ferrara in May 2012. It includes some historical aspects, the spectral properties of the 2-body and 3-body Schrödinger operators applied to mesons and baryons, the link between meson and baryon spectra, the role of flavour independence, and the speculations about stable or metastable multiquarks. The analogies between few-charge systems and few-quark bound states will be underlined.

Contents

1 Prelude: few charge systems in atomic physics

Si parva licet componere magnis [1]
Virgil

1.1 Introduction

The spectrum of few-charge systems was among the very first applications of quantum physics. The Bohr–Sommerfeld rules explain the energy levels of the hydrogen atom (p, e^-), and can be easily extended to any (m_1^+, m_2^-) pair with arbitrary masses. The solution of the three-body problem for (m_1^+, m_2^-, m_3^-) turned out less obvious, and required the Schrödinger equation, and the associated variational methods. It also revealed some surprises, with the stability imposing constraints on the masses m_i.

The stability of the positronium molecule (e^+, e^+, e^-, e^-), or any similar system with equal masses (in the limit where annihilation is neglected) was suggested by Wheeler in 1945 [1] and proved in 1947 [2], but the (indirect) experimental evidence was published only in 2007 [3]. This indicates how patient one should be when predicting novel structures.

The quantum mechanics of few-charge systems was a great source of inspiration for building the quark model, in its minimal non-relativistic version. Amazingly, some techniques developed to extrapolate into flavour space the few-quark spectra with flavour-independent forces turned out useful to understand the stability patterns in atomic physics. I refer to [4] for a review about few-charge systems. Here, I will just stress a few points that are connected to the quark model.

1.2 The atomic two-body problem

1.2.1 Central potential

Out of the Hamiltonian

$$\frac{\boldsymbol{p}_1^2}{2\,m_1} + \frac{\boldsymbol{p}_1^2}{2\,m_1} - \frac{e^2}{r_{12}}\,, \tag{1}$$

the centre of mass can be removed and one is left with the intrinsic Hamiltonian describing the relative motion, which reads

$$H = \frac{\boldsymbol{p}^2}{2\,\mu} - \frac{e^2}{r}\,, \tag{2}$$

where μ is the reduced mass, given by $\mu^{-1} = m_1^{-1} + m_2^{-1}$, and $\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$ and $\boldsymbol{p} = (\boldsymbol{p}_2 - \boldsymbol{p}_1)/2$ are conjugate variables for the relative motion.

The Hamiltonian H has seemingly two parameters, the reduced mass μ and the strength e^2, but it can be rescaled. It is sufficient to solve once for ever the universal spectral problem

$$h = -\Delta - r^{-1}\,, \tag{3}$$

[1] if it is allowed to compare small things with great

and to apply a simple factor $2\,\mu\,e^4$ to the eigenvalues and $(2\,\mu\,e^2)^{-1}$ to the distances to recover the actual energies and wave-functions of (2).

There is a similar universality for all harmonic oscillators $\boldsymbol{p}^2/m + K\,\boldsymbol{r}^2$, and more generally for all power-law interactions [5]

$$H = \frac{\boldsymbol{p}^2}{2\,\mu} + g\,\epsilon(\alpha)\,r^\alpha\ , \tag{4}$$

where $g > 0$, ϵ is the sign function, to ensure that the force is attractive, and α is not too large if negative, otherwise the Hamiltonian would not be bounded from below.

A common feature of the Coulomb and Hookes potentials in three dimensions, is that they support an infinite number of bound states, however weak is the strength of the potential. This contrasts with short-range potentials such as the Yukawa interaction $-g\,\exp(-b\,r)/r$ of nuclear physics, which requires a minimal strength g to achieve binding. Similarly, the effective potential between neutral atoms or the effective potential between two hadrons are of short-range nature, and thus do not necessarily support bound states even when they are attractive.

The Coulomb Hamiltonian (3) has a well-known spectrum, with a discrete part $\epsilon_{n,l} = -1/[4\,(n+\ell)^2]$, where $n = 1, 2, \ldots$ is the radial number, and $\ell = 0, 1\ \ldots$ the orbital momentum ($n+\ell$ is the principal quantum number). There is also a continuum for $\epsilon \geq 0$.

1.2.2 Spin-dependent corrections

The Coulomb interaction can be derived from Quantum ElectroDynamics (QED) in the non-relativistic limit, and in the approximation of the lowest order in the coupling constant, which corresponds to one-photon exchange.

There are several interesting corrections, which have been probed successfully. In particular, the vector nature of the photon gives very characteristic spin corrections.

For $\ell \geq 1$, there are spin-orbit and tensor terms which contribute to the fine structure: levels with the same orbital momentum ℓ but different coupling of ℓ to the spins have slightly different energies.

For $\ell = 0$, there is spin–spin term, which splits the spin-singlet form the spin triplet states. It reads

$$V_{ss} = \frac{e^2}{m_1\,m_2}\,\frac{2\,\pi}{3}\,\delta^{(3)}(\boldsymbol{r})\,\boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2\ , \tag{5}$$

in its most simplified form, to be treated at first order in perturbation theory, using the wave functions generated by the central potential $-e^2/r$. The short-range character, and the dependence on the masses should be kept in mind when an analogue will be proposed for the quark–antiquark interaction. This interaction is responsible for the shift between the ortho- and para-hydrogen states and the transition between them produces the famous 21 cm line, whose gravitational red-shift and Doppler shift gives valuable information in astrophysics. The analogue will be measured at CERN for antihydrogen, to probe the matter–antimatter symmetry.

Note that in the case of the positronium atom, the hyperfine splitting also receives some contribution from the annihilation diagrams.

1.3 Three-unit-charge ions

The best known 3-body system is atomic physics is the neutral Helium atom, (α, e^-, e^-), but its stability is obvious since once the first electron is attached to the α particle, there remains some long-range attraction to bind the second electron.

The binding of $\mathrm{H}^- = (p, e^-, e^-)$ is less obvious, as the second electron feels a neutral system at large distances. Unlike the case of the Helium atom, neither an effective-charge ansatz $\psi(r_2, r_3) = \exp(-a(r_2 + r_3))$, nor a more general Hartree wave function $f(r_2)\, f(r_3)$ suffices to demonstrate the binding variationally. But a more elaborate wave function does. For a review on two-electron systems, see, e.g., [6].

More interesting, perhaps, is the dependence upon the masses [4]: the molecular hydrogen ion $\mathrm{H_2}^+ = (p, p, e^-)$, is very stable, with a variety of excitations, the hydrogen ion $\mathrm{H}^- = (p, e^-, e^-)$ and the positronium ion $\mathrm{Ps}^- = (e^+, e^-, e^-)$ are bound by a small margin, but the "protonium ion" $(p, \bar{p}, e^\pm)$ is not stable. (Here p and $\bar{p}$ simply denotes the mass and the Coulomb charge; any hadronic interaction is neglected.)

There is an increase of stability near the symmetry line $m_2 = m_3$ of $(m_1^\pm, m_2^\mp, m_3^\mp)$. Schematically, the system has there two threshold configurations $(m_1^+, m_2^-) + m_3^-$, and $(m_1^+, m_3^-) + m_2^-$, which are degenerate, and thus interfere optimally to achieve the best binding.

This mass dependence is sometimes acute. For instance, a fictitious H^- with one electron heavier by about 10% would not be be stable. This is the case, e.g., for the (very) exotic (p, μ^-, π^-) ion.

1.4 Three-body exotic ions

It is sometimes believed that H^- has no excited bound state. This is both true and false. This is true if you define a bound state as lying below the lowest threshold, which is made of $\mathrm{H} = (p, e^-)$ in the 1S state and an isolated electron. However, there is a level with total orbital momentum $L = 1$ and parity $+1$, that cannot break into $\mathrm{H}(1S) + e^-$ without involving spin forces and radiative effects. For this state, the lowest threshold for spontaneous dissociation consists of $\mathrm{H} = (p, e^-)$ in the 2P state and an electron. A bound state lying below this threshold exists for $\mathrm{H}^- = (p, e^-, e^-)$, but it exhibits even a more striking mass dependence when the constituents are modified [7]. Allowing a very small mass difference between the two electrons (less than 1%) or replacing the proton by a positron spoils the binding. On the other hand, if the masses are inverted, i.e., for the unnatural-parity state of $\mathrm{H_2}^+ = (e^-, p, p)$, a comfortable binding is observed.

1.5 Molecules with four unit charges

The best known case corresponds to the hydrogen molecule $\mathrm{H}_2 = (p, p, e^-, e^-)$, bound well below the threshold for dissociation into two hydrogen atoms, with a variety of excitations. There are also variants with one or two protons replaced by an isotope. The most suited framework for studying H_2 is the Born–Oppenheimer approximation. For a given position of the protons, the electronic energy is computed, and interpreted as an effective proton–proton interaction supplementing their direct electrostatic repulsion. The Schrödinger equation is then solved for the two-body problem of the protons and generates the ground state and its radial and orbital excitations. Another set of levels correspond to the electron cloud being in an excited state. (Later the analogue will be the hybrid hadrons, with the gluon field linking the quarks being excited). Note that one hardly treats the hydrogen molecule as a "di-proton" bound to a "di-electron".

In atomic physics and ab-initio chemistry, one usually starts from the limit where the proton and other nuclei are infinitely heavy, and then their finite mass can be treated as a correction, through the Hughes–Eckart terms or similar. In 1945, Wheeler [1] addressed the somewhat opposite issue of a proton with the same mass as the electron, i.e., the problem of the stability of the positronium molecule, (e^+, e^+, e^-, e^-) (in the limit where annihilation is neglected). In 1946, Ore performed an elaborate 4-body calculation, and concluded that such a configuration is hardly stable [8]. But the following year, Hylleraas and the very same Ore published an elegant analytic proof of the stability of the positronium molecule [2]. It took 60 years to obtain at last an indirect experimental evidence of the existence of this molecule [3].

Meanwhile the dependence upon the masses was studied in some details. In Refs. [9–12], it was shown how (M^+, M^+, m^-, m^-) gets more binding, as compared to its threshold, when the mass ratio departs from $M/m = 1$. On the other hand, it was observed that when allowing for two different masses in another way, (M^+, m^+, M^-, m^-), stability is lost for $M/m \gtrsim 2.2$ (or, $M/m \lesssim 1/2.2$), because the molecule cannot compete any more with the lowest threshold $(M^+, M^-) + (m^+, m^-)$ [13, 14].

Many less symmetric configurations are bound, such as the positronium hydride $\mathrm{PsH} = (p, e^+, e^-, e^-)$, or any configurations in which two particles have the same charge and the same mass, that is to say, (a^+, b^+, c^-, c^-), whatever are the masses of a and b [15].

The lessons, to be kept in mind when arriving at the section on multiquarks, are:

- the role of the masses is important. Look at (a^+, b^+, c^-, d^-) vs. its lowest threshold supposed to be $(a^+, c^-) + (b^+, d^-)$. For both the 4-body system and its threshold, the potential energy $\sum g_{ij}\, r_{ij}^{-1}$ has a cumulated strength $\sum g_{ij} = -2$. So, there is no obvious reason why a molecule should lie lower that two atoms. Hence any favourable breaking of symmetry in the kinetic energy is welcome. Conversely, any unfavourable breaking can spoil the binding.

- the four-body problem is delicate, even when the interaction is known perfectly,

- one should be patient to see the experimental confirmation of theoretical predictions of exotic states.

2 A brief historical survey

History is Philosophy teaching by examples[2]
Thucydid

2.1 Prehistory

There are several books relating the birth of particle physics, with entertaining anecdotes. Segrè, for instance, was an acute observer [16]. A very comprehensive review is given by Pais [17]. For the nuclear forces, see [18]. Books and reviews devoted to the quark model include [19–25]. There are also reviews covering baryons [26–28] or mesons [29–31], in particular the synthesis by the "quarkonium working group" [32,33].

After the Rutherford experiment, indicating how compact is the atomic nucleus, and the discovery of the neutron by Chadwick, it became necessary to understand how the nucleus is built out of protons and neutrons. The mechanism of Yukawa: the exchange of a massive boson, turned out the be successful, with the discovery of the pion at Bristol in 1947.

Among the theoretical activity stimulated by the Yukawa model, two points at least deserve attention.

1. The mass of the Yukawa boson is constrained by the ratio of the 3-body to the 2-body binding energy, as stressed by Thomas in a celebrated paper [34], who anticipated what is known today as "Borromean binding", and, more generally, "Efimov physics".

2. At first sight, three potentials are needed to build the nucleus: proton–proton, proton–neutron and neutron–neutron, and it is natural to seek for some simplification. A tempting scenario is that where solely the proton–neutron interaction exists, but this is clearly contradicted by the data on proton scattering. What eventually prevailed is isospin symmetry. With the proton and the neutron forming a doublet, there are only two potentials in the limit where isospin is conserved, one for total isospin $I = 0$, and another one for $I = 1$.

2.2 Early hadrons

The pion was discovered in 1947 and seen in three charge states, π^+, π^0, and π^- which form an isospin triplet. Hence in 1947, we had 5 hadrons: 2 nucleons and 3 pions. But already two more were expected, as the existence of antimatter was (not so easily) predicted as a consequence of the Dirac equation, and the positron was discovered in cosmic rays by Anderson. The antiproton and the antineutron were anticipated as well. A dedicated accelerator was built at Berkeley, the Bevatron, and the antiproton was, indeed, discovered in 1955, and the antineutron shortly after.

[2] According to Michel Casevitz, the sentence is not by Thucydid, but a British commentator

With 7 hadrons, 2 nucleons, 2 antinucleons, and 3 pions, the world of hadronic physics would be reasonably sized, and one could envisage to work on the interaction among these few hadrons. However, several complications occurred almost simultaneously.

First, the Yukawa picture of nuclear forces, though very efficient for the long-range part, faced difficulties at shorter distances. More attraction was needed, and also some spin-orbit component, that neither pion-exchange or iterated pion-exchange were able to provide. The work by Breit, among others, was remarkable [35]. For further details, see, e.g., the review [18]. An explicit scalar exchange (call it σ or ϵ) and an explicit vector exchange were needed. While the former was about isospin independent, and thus provided by the exchange of an isoscalar scalar meson, the latter was sought to be different in np vs. pp scattering, and thus called for both an isoscalar and an isovector vector meson. The ω and ρ were thus predicted!

Second, the interaction of pions with nucleons was shown to produce new particles, nucleon resonances, in particular the $\Delta(1232)$, which has isospin 3/2, i.e., exists in four possible electric charges. Similarly, proton–nucleon or proton–nucleus scattering, or proton–antiproton annihilation were able to produce several new mesons, the ones desired to improve the theory of nuclear forces, and others. These hadrons are not stable, with for instance $\Delta \to N+\pi$ or $\rho \to \pi+\pi$, but were named hadrons as well, baryons or mesons.

The Δ, at first sight, appears as a consequence of the interaction between π and N, and the ρ as a resonance of the $\pi\pi$ interaction. Chew and his collaborators generalised this scenario, and suggested the concept of "bootstrap" or "nuclear democracy" [36,37]: everything is made of everything, and any hadron is both a building block and the result of the interaction of the other hadrons. For instance, Δ is made of $N\pi$, $N\pi\pi$, etc., and, as well N is made of $\Delta\pi$, etc. This gives an infinite set of coupled equations, of which it was hoped one could extract a finite set as a first tractable approximation to the spectrum and to the dynamics. The success was, however, extremely limited. Ball, Scotti and Wong, for instance, stressed that describing mesons as resulting of the nucleon–antinucleon interaction hardly gives the observed "exchange degeneracy" (named after the phenomenology of high-energy scattering), the property that an isoscalar and an isovector mesons with the same quantum numbers have very often the same mass [38]. In the baryon sector, the next state after the Δ with isospin $I = 3/2$ and spin $J = 3/2$ was sometimes predicted to have $I = 5/2$ and $J = 5/2$!

Third came strangeness. New particles were observed in the 50s and 60s, decaying weakly though massive enough to decay to existing particles, and produced by pairs with strict rules: K^+ together with Λ for instance, but never K^- together Λ. A new quantum number, strangeness S, was introduced to summarise the properties of these new particles: strangeness is conserved in production processes by strong interaction, and thus $\Lambda(S = -1)$ can be produced in association with $K^+(S = 1)$, but not K^- which has $S = -1$. On the other hand, strangeness is not conserved in the decay by weak interaction, as $\Lambda(S = -1) \to N + \pi$, or $\Lambda \to p + e^- + \bar{\nu}$. The weak interaction of strange particle was beautifully linked to that involved in ordinary beta decay.

2.3 Generalised isospin

There is of course the exception of the π meson, with a mass of about 0.14 GeV, significantly lighter than the mass of the K, about 0.49 GeV, and the exception of light scalar mesons with long-standing questions about their structure. Otherwise, one observes that strange particles do not differ much form the non-strange ones. For instance, the mass of the Λ baryon is about 1.1 GeV, just slightly above that of the nucleon at 0.94 GeV, and the mass of the K^*, 0.89 GeV, is close to that of the vector mesons ρ and ω, about 0.78 GeV. It was thus tempting to put strange and non-strange particles in multiplets generalising isospin. Since isospin is built on the SU(2) group, the minimal extension is SU(3). Later, it was renamed $\mathrm{SU(3)_F}$, to differentiate it from the SU(3) group associated with colour.

A specific model was proposed by Sakata [39], with (n, p, Λ) as the building blocks of matter, and mesons as baryon–antibaryon pairs. We already mentioned that this picture of mesons is difficult to accommodate with the long-range baryon–baryon interaction as given in the Yukawa picture. But the Sakata model had also problems with baryons. There are too many baryons with low mass. Take for instance the lowest baryons with spin $J = 1/2$. Besides p, n, and Λ, there is a triplet de singly-strange ($S = -1$) baryons $(\Sigma^+, \Sigma^0, \Sigma^-)$ of mass about 1.3 GeV, and a pair of doubly strange ($S = -2$) baryons (Ξ^0, Ξ^-) with mass about 1.5 GeV. In the Sakata model, they should belong to higher representation, and, in a dynamical picture, contain an additional baryon–antibaryon pair. This is hard to believe.

To take care of this problem, Ne'emann and Gell-Mann suggested to keep the SU(3) group as the basic symmetry, but to put the known $J = 1/2$ baryons in an octet representation. This is the famous "eightfold way" [40]. The group SU(3) has eight generators, instead of three for SU(2) ($I_\pm$ and I_3). Each multiplet can be characterised by the dimension of the representation (this generalises the $2\,I + 1$ multiplicity for SU(2)), and two generators that commute, which are usually taken as I_3 and strangeness S, or equivalently the *hypercharge* defined as $Y = b + S$, where b is the baryon number. SU(3) is

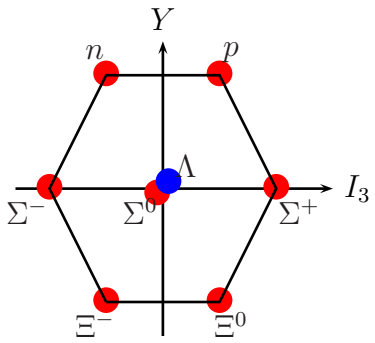

Figure 1: Octet of the lowest spin $J = 1/2$ baryons

rather good symmetry. Its breaking can be described by simple terms, treated at first order, which are proportional to some generators of the group. More elaborate mecha-

nisms were proposed for breaking SU(3). One example is the famous Gell-Mann–Okubo formula for the baryon masses

$$M = M_0 + a\,Y + b(I(I+1) - Y^2/4)\,, \tag{6}$$

from which one gets

$$2(N + \Xi) = 3\,\Lambda + \Sigma\,, \tag{7}$$

where each particle stands for its mass, in surprisingly good agreement with experiment.

2.4 The success of the eightfold way

When the eightfold way was proposed, only 9 baryons with spin $J = 3/2$ were known, with a low mass: four Δ of charge ranging from -1 to $+2$, thre Σ^* with strangeness $S = -1$ and two Ξ^* with strangeness $S = -2$. At the 1962 Rochester Conference held in Geneva, Gell-Mann pointed out they would fit very well a decuplet representation of SU(3), provided the last member does also exist. He named it Ω^-, as a kind of ultimate achievement, in the biblical sense. The masses of the 9 existing members being observed to grow linearly with strangeness, i.e., following the pattern

$$\Sigma^* - \Delta = \Xi^* - \Sigma^*\,, \tag{8}$$

known as the "equal spacing rule of the decuplet", it was tempting to extrapolate to include $\Omega^- - \Xi^*$ in the equality, thus predicting the mass of the Ω^- at about 1.67 GeV. Some physicists were sceptical about the possibility of producing and detecting easily the Ω^-. It was nevertheless seen in an experiment at Brookhaven led by Nick Samios, with exactly the mass predicted by Gell-Mann. This was at the end of 1963, and published in 1964 [41], which otherwise was one of the best vintages ever in Burgundy!

2.5 The fundamental representation: quarks

The pseudoscalar mesons (π, η, K, $\bar{K}$) were accommodated in an octet and a singlet. As there is presumably mixing between the singlet and the isoscalar member of the octet, it became customary to talk about "nonet". The same holds for the vector mesons. See Fig. 3.

We thus had already in the hadronic world octets, singlets and decuplets, but no triplet, which corresponds to the fundamental representation! Gell-Mann proposed that the fundamental representation is populated by three yet not discovered – or hypothetical – particles. He was of course fully aware that any representation can be built by combining the fundamental representation, 3, and its conjugate, $\bar{3}$, in the same way that any value of the spin can be built by adding elementary spins $1/2$.

The word "quark" was taken from the sentence "Three Quarks for Muster Mark" in Joyce's Finnegans Wake. In another context (see next subsection), the word "ace" was

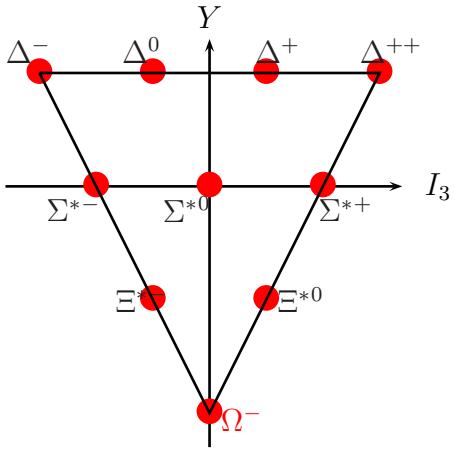

Figure 2: Decuplet of the lowest spin $J = 3/2$ baryons. The Ω^- was missing till its discovery in 1964.

suggested, but did not prevail. The individual quarks were sometimes named p, n and λ, in reference to the Sakata model, but quickly the naming scheme u, d, s was adopted by the community.

The properties of the quarks are summarized in Table 1 and Fig. 4

Table 1: Properties of the quarks

q	b	I	I_3	Y	S	Q
u	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{2}{3}$
d	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{1}{3}$
s	$\frac{1}{3}$	0	0	$-\frac{2}{3}$	-1	$\frac{2}{3}$

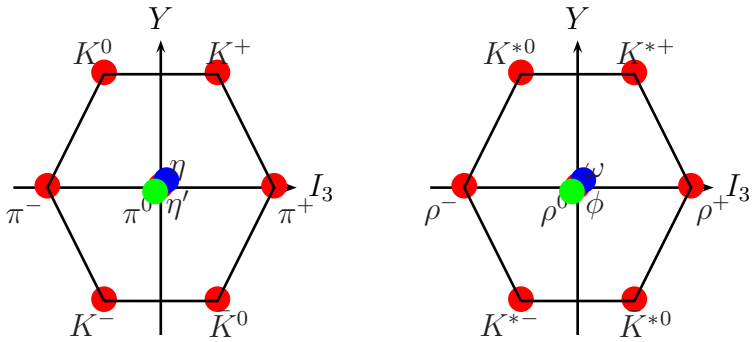

Figure 3: Octet and singlet of pseudoscalar (left) and vector (right) mesons.

The basic reduction of products of representations that are important for the quark model are

$$3 \times 3 \times 3 = 1 + 8 + 8 + 10\,, \quad 3 \times \bar{3} = 1 + 8\,. \tag{9}$$

Historians could debate endlessly whether the pioneers considered quarks as a handy mathematical tool to build the representation of SU(3), or had already in mind a physical interpretation of the quarks as the constituents of the hadrons. Anyhow, this was one of the major breakthroughs in physics.

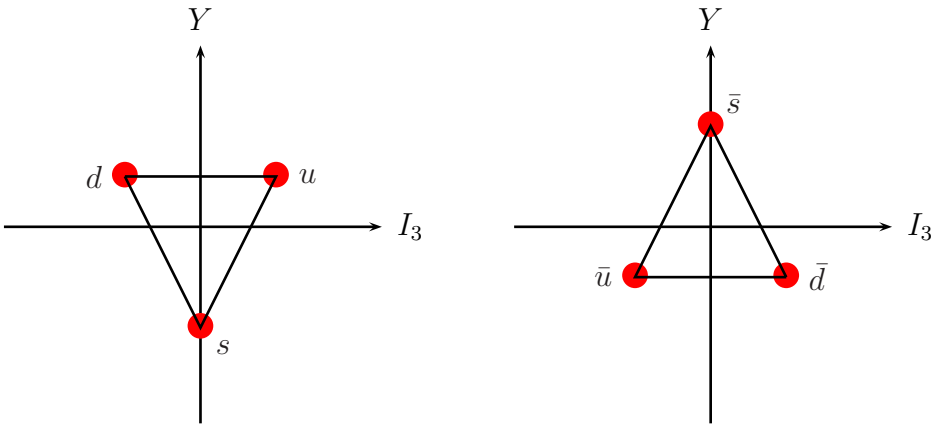

Figure 4: Triplet of quarks and antitriplet of antiquarks

2.6 The OZI rule

Another approach was followed by Zweig. See, e.g., his recollection at the Conference Baryon80 [42] or at the celebration of Gell-Mann's 80th birthday [43].

The ϕ meson, of mass about 1.02 GeV, was discovered in 1962 [44, 45]. It is an isoscalar, vector meson, like the ω, but with peculiar decay patterns. While it could easily decay into pions, it prefers the $K\bar{K}$ channels, for which the phase-space is meagre (the K has a mass of about 0.49 GeV).

Zweig's explanation is that this favoured decay is dictated by ϕ meson content. He named the constituent "aces", but we will call them quarks to conform with the current usage. The idea is that the decay preferentially keeps the existing content. In the modern language, the ϕ decay is described by the diagrams of Fig. 5.

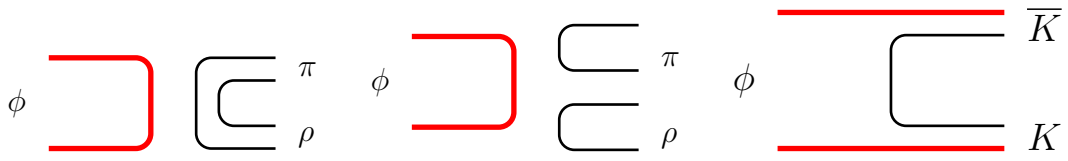

Figure 5: Decay of the ϕ meson: Zweig forbidden (left, centre) or allowed (right)

He thereby invented the "Zweig rule", also called Okubo–Iizuka–Zweig (OZI), or (A–Z) rule since many authors contributed, from Alexander to Zweig. For a review, see, e.g., [46]. As we shall see shortly, a nice surprise was that the rule works even better for heavier quarks.

The rule governing the ϕ decay was extended to other decays and to reaction mechanisms. Quark lines should not start from and end in the same hadron, i.e., disconnected diagrams are suppressed. Instead quark lines should better link two hadrons in the initial or final state.

The OZI rule then got variants. It is sometimes argued that the dominant processes correspond to planar diagrams, while non-planar (but still connected) diagrams are suppressed. For instance, there are dozens of measured branching ratios for antinucleon–nucleon annihilation at rest or in flight. In Fig. 6, the rearrangement diagram (left) clearly keeps the initial constituents, while the diagram on the right is more planar. Clearly, the former does not produce enough kaons, while the second predicts much more kaons than observed.

Note that antiproton annihilation is an indirect evidence for quarks, but this was understood much after the first measurements. If guidance is sought from QED, annihilation has a rather small cross section, because it is a very short-range process. The results obtained at Berkeley for the elastic and annihilation cross-sections of antiproton scattering on nucleons indicated that the latter is larger. To reproduce these results with an empirical (complex) potential, one needs a large size for the annihilation part, about 0.8 fm or more. This was a puzzle. One now understands that the proton and the antiproton are composite, with a size of the order of 0.5–1 fm. Hence, when they overlap, they can rearrange their constituents into quark–antiquark pairs. This is similar to the rearrangement collisions in molecular physics, but has little to do with e^+e^- annihilation in QED.

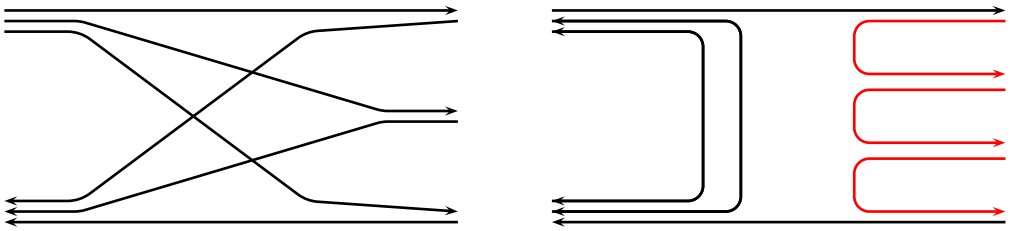

Figure 6: Possible diagrams contributing to baryon–antibaryon annihilation. The hierarchy of these diagrams requires an extension of the Zweig rule that remains a little controversial.

2.7 First quark models

Greenberg, in a celebrated paper [47], tried to understand the structure of baryons made of quarks. In this paper, he suggested a kind a harmonic oscillator as a first approx-

imation to describe the quark motion. He addressed the problem of the statistics and suggested a kind of para-statistics, that eventually became the colour degree of freedom.

The work of Dalitz was done almost simultaneously. In the summer of 1965, and in particular in his lectures at the School of Les Houches [48], he constructed his first version of the harmonic oscillator model of baryons. As Greenberg, he faced the problem of the statistics of the quark.

Dalitz's work was the starting point of a series of studies about baryons in the harmonic oscillator model with contributions by Horgan, Hey, Kelly, Reinders, Gromes, Stamatescu, Stancu, Cutosky, etc., culminating with Isgur and Karl. Potential models not based on harmonic confinement were proposed somewhat later. The references will be given in the section on baryons.

Note also the contribution by Becchi and Morpurgo about the possibility of describing hadrons made of quarks in a non-relativisitic approximation [19, 20, 49, 50].[3]

2.8 Heavy quarks

The physics of kaons has been very stimulating along the years: strangeness led to the quarks, the $\theta - \tau$ puzzle led to parity violation and to $K^0\bar{K}^0$ mixing, whose detailed scrutiny revealed CP violation [17]. Another problem in the weak decay of kaons inspired Glashow, Illiopoulos and Maiani [51], who predicted a fourth quark, named "charmed quark" and abbreviated as c, whose mass should not be too high. In some processes, diagrams involving a u and diagrams involving a c cancel out. This is the GIM mechanism.

A new spectroscopy was thus predicted, with charmed mesons such as $(c\bar{u})$, or charmed baryons such as (csu), double-charm baryons, etc. See, e.g., Gaillard, Lee and Rosner [52]. Of course, $(c\bar{c})$ mesons were predicted as well.

However, when in November 1974 (this was sometimes called the October revolution), the J/ψ was discovered simultaneously at SLAC and Brookhaven, and the ψ' shortly after at SLAC, they were not immediately recognised as $(c\bar{c})$, i.e., charmonium. The surprise was that they are extremely narrow resonances. We now understand that the Zweig rule works better and better when the quark become heavier. The spectrum of charmonium was completed with various P state, the $\psi'' = \psi(3770)$ which is a D state, and after some time, with the spin-singlet states.

Charmed mesons [53] and baryons [54] were discovered as well, and this sector is now rather rich, tough the double- and triple-charm baryons still await discovery.

The charmonium gave a decisive impulse to the quark model in the meson sector. Thanks to Regge and others, we had already an idea about sequences of mesons with increasing spin J. In the quark model, this corresponds to orbital excitations of the quark–antiquark motion. With charmonium, the new feature is that *i)* the levels are better seen, since the lowest states are narrow, *ii)* there is a clear evidence for the radial

[3] I thank Pr. Morpurgo for a correspondence related to this subject

degree of freedom. Explicit quark models were developed to describe the $(c\bar{c})$ spectrum. They will be reviewed in Sec. 3.

At the time of the discovery of charm, in the 70s, there were already speculations about a symmetry between quarks and leptons. The light quarks (u, d) are the partners of (e^-, ν_e). The strange and charmed quarks belong in the family of (μ^-, ν_μ).

Note that the leptons are ahead. The μ^- was discovered in 1936,[4] and was completely unexpected ("who ordered the muon? asked Rabbi). The τ lepton was discovered at SLAC in 1977. The partners of the (τ^-, ν_τ) pairs were thus anticipated and named (t, b), as "top" and "bottom". And a spectroscopy of hadrons containing b or t, or both, was predicted. However, at a Conference in Hamburg, a German minister who had some knowledge of English, suggested to replace bottom by "beauty". And t was renamed "truth quark", but this is not very often used.

In 1977, Leon Lederman, who missed the charmonium by a small margin, did an experiment similar to Ting's, but with a more powerful beam and an improved detector, and announced in 1977 the discovery of the Υ and Υ' particles, immediately interpreted as $(b\bar{b})$ bound states [55, 56]. We shall come back in the next sections about the role of flavour independence. Just a word here. Lederman noticed that, within the precision of his measurement, $\Upsilon' - \Upsilon \simeq \psi' - J/\psi$, and submitted to local theorists the question whether there exists a potential such that changing the reduced mass keep the spacings ΔE unchanged (remember that $\Delta E \propto m$ for the Coulomb potential and $\Delta E \propto m^{-1/2}$ for the harmonic oscillator). Quigg and Rosner found that the logarithmic potential has this property for all spacings [57]. In fact, the logarithmic potential was already used in empirical speculations about the not-yet-discovered bottomonium, but this property was not stressed clearly enough [58].

Predictions for $(t\bar{t})$ were revised continuously, as the mass of the top was pushed higher and higher by the negative results of the experimental searches. It was then stressed (see, e.g., [59] for an early review) that if the top quark is very heavy, it will decay before hadronising.

2.9 Confirmation

On the way from the early days of the quark model to the recent state of art, there are many beautiful and decisive contributions that unfortunately, I cannot review here, due to the lack of time. I will just list of few of them: deep inelastic scattering, the parton model, the MIT bag model, and its many variants, QCD, its property of asymptotic freedom, lattice QCD, QCD sum rules, etc., etc.

[4] In 1947, the discovery of the pion was confused by the decay of pions into muons

3 The quark–antiquark model of mesons

I married them
Friar Laurence, Romeo and Juliet

3.1 Introduction

The quark model of mesons has not been developed "from the bottom up", but mainly "from the charm down to light sector, and from the charm to the beauty quark".

At the end of 1974, when the new particles seen at Brookhaven and SLAC were identified as $(c\bar{c})$ bound states, explicit models were proposed to calculate the spectrum and the radiative transitions [60–63] . The potential proposed in [63] is know as the funnel potential or Cornell potential, and reads,

$$V(r) = -\frac{a}{r} + b\,r + c \tag{10}$$

A new era of meson spectroscopy was opened, with explicit calculations of the meson properties in models which were first guessed empirically and are now more seriously inspired by QCD.

3.2 Quantum numbers

The total angular momentum J results from the coupling of the spin S of the quarks and their orbital momentum ℓ, from which one also gets the parity $(-1)^{\ell+1}$ and charge conjugation $C = (-1)^{\ell+\mathsf{S}}$. The lowest states are given in Table 2.

Table 2: Quantum numbers of the lowest quarkonium states

$^{2\mathsf{S}+1}\ell_J$	$^1\mathrm{S}_0$	$^3\mathrm{S}_1$	$^1\mathrm{P}_1$	$^3\mathrm{P}_0$	$^3\mathrm{P}_1$	$^3\mathrm{P}_2$	$^1\mathrm{D}_2$	$^3\mathrm{D}_1$	$^3\mathrm{D}_2$	$^3\mathrm{D}_3$
J^{PC}	0^{-+}	1^{--}	1^{+-}	0^{++}	1^{++}	2^{++}	2^{-+}	1^{--}	2^{--}	3^{--}

Comments are in order

- some sets of quantum numbers are absent. For instance, a state with $J^{PC} = 1^{-+}$ would be exotic,
- some J^{PC} occur twice. There is the possibility of, e.g., $^3\mathrm{S}_1 - {}^3\mathrm{D}_1$ mixing, as for the deuteron,
- the above quantum numbers are repeated for the radial excitations, labelled with the radial number n. In the literature, one can find either $n = 0, 1\ldots$ as in the familiar one-dimensional oscillator, or $n = \ell + 1, 2, \ldots$ as in the Coulomb problem. In the latter case, the first P state is labelled 2P, the first D state 3D. We shall adopt $n = 1, 2, \ldots$, i.e., a counting where the radial wave function has $(n-1)$ nodes.

3.3 Spin averaged spectrum

With a potential such as (10) which is central, without spin dependence, the energy depends only on ℓ and n. The radial equation reads

$$-u''(r) + \frac{\ell(\ell+1)}{r^2}\,u(r) + m\,V(r)\,u(r) = m\,E\,u(r)\ , \tag{11}$$

where $m = m_c$ is the mass of the charmed quark in the case of quarkonium, and otherwise m is twice the reduced mass. For a pure Coulomb interaction, this equation can be rescaled to $-v''(r) + (\ell(\ell+1)/r^2 - 1/r - \epsilon)v(r) = 0$, with eigenenergies $\epsilon = -1/(4\,n^2)$, with $n = \ell+1, \ell+2, \ldots$. For a purely linear interaction, another rescaling leads to $-v''(r) + (\ell(\ell+1)/r^2 + r - \epsilon)v(r) = 0$, and in the case where $\ell = 0$, it reduces to a shifted Airy equation: the eigenenergies are the negative of the zeros a_i of the Airy function $\mathrm{Ai}(x)$, and the n^{th} eigenfunction is just the very same Airy function shifted at a_n, $v_n(r) \propto \mathrm{Ai}(r + a_n)$, the normalised version being

$$v_n(r) = \frac{\mathrm{Ai}(r+a_n)}{\mathrm{Ai}'(a_n)}\ . \tag{12}$$

If both the Coulomb and the linear terms are present, one can rescale to a one-parameter problem, which can be chosen as

$$\left[-\Delta - \frac{\lambda}{r} + r - \epsilon\right]\psi(\boldsymbol{r}) = 0\ , \tag{13}$$

where λ can be expressed in terms of the quark mass m and the strength parameters a and b of the Cornell potential.

If the two-body problem is solved, one can tune the parameters to reproduce the low levels of charmonium. This was done by several groups in the late 70s, and the authors were able to predict the missing states.

Note that this exercise did not provide with a sharp determination of the parameters, e.g., a, b and c in (10), and the mass m_c of the charmed quark. Additional constraints were used, such as the leptonic widths, and the rates of the γ-transitions. Still some flexibility was allowed, and, when the first bottomonium levels were found, the game became more challenging: to reproduce simultaneously the $(c\bar{c})$ and the $(b\bar{b})$ spectra.

Indeed, even so the interquark potential was not derived from QCD in early quarkonium phenomenology, it was assumed that it is universal, or "flavour independent". In QCD the gluons couple to the colour. Hence it is reasonable to assume that the potential is flavour independent.

We already mentioned the logarithmic solution for $V(r)$ [57]. Then the radius scales as $m^{-1/2}$, and the square of the wave function at the origin, $|\phi(0)|^2$, that enter several decay widths, scale as $m^{-3/2}$. As $\ln r = \lim(r^\alpha - 1)/\alpha$ as $\alpha \to 0$, a generalisation consists of a power-law interaction $V(r) = A\,r^\alpha + B$. Martin used this functional form to fit the quarkonium levels, and obtained a rather good fit [64] with a small value of α, i.e., a potential close to the logarithmic one. See, also, [65].

A typical charmonium potential is shown in Fig. 7, as well as the reduced radial wave function $u(r)$ for the 1S and 2S states. In units of GeV for V and GeV^{-1} for r, it reads $V(r) = -.4/r + 0.2\,r - 0.35$. It is just for illustration purpose, without any attempt to achieve the best fit.

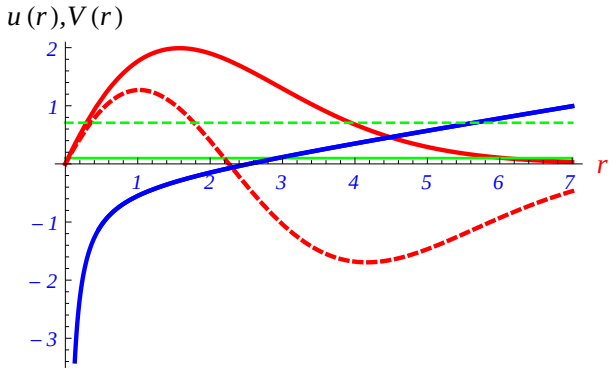

Figure 7: Simple central potential $-0.4/r + 0.2\,r - 0.35$ and two first S-wave levels for a quark mass $m_c = 1.5$. Units are GeV for V, GeV^{-1} for r, and arbitrary for the reduced radial functions $u(r)$.

The corresponding results are listed in Table 3. This should be considered as a kind of 0th order starting point which could be refined by tuning the parameters, fitting some higher levels, and including some decay properties in the constraints.

Table 3: Rough fit to the spin-averaged levels of quarkonium (for 1D, the experimental value corresponds to ψ'' which is ^{3}D$_1$, and the state seen by CLEO [66] and BABAR [67], which is presumably ^{3}D$_2$

	$c\bar{c}$				$b\bar{b}$				
	1S	2S	1P	1D	1S	2S	1P	1D	2P
Model	3.07	3.68	3.48	3.78	9.47	9.99	9.87	10.11	10.23
exp.	3.07	3.67	3.52	3.77	9.44	10.01	9.89	10.16	10.26

In Fig. 8 are shown the levels of bottomonium using the simple potential $-0.4/r + 0.2\,r - 0.35$ and a mass $m_b = 4.5$ GeV. For the very beginners, its a good exercise to reproduce these numbers. The hierarchy of the excitations corresponds to the observation.

In Fig. 9, we supplement the previous figure by the charmonium levels, computed with the same simple potential. The 1S levels are arranged to coincide. The potential is tuned to produce about the same spacing for the lowest states. But for the higher states, the spectrum becomes dominated by the linear part of the interaction, and the spacing is significantly higher for $(c\bar{c})$ than for $(b\bar{b})$.

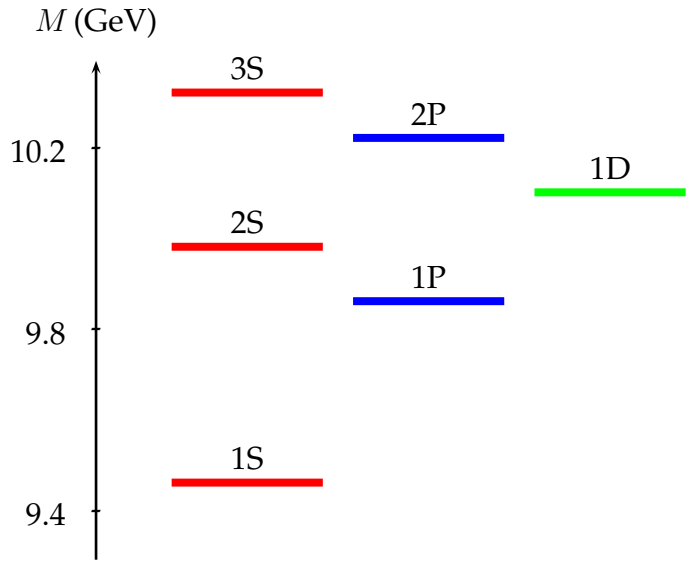

Figure 8: Predictions for the $b\bar{b}$ spectrum with a very simple potential.

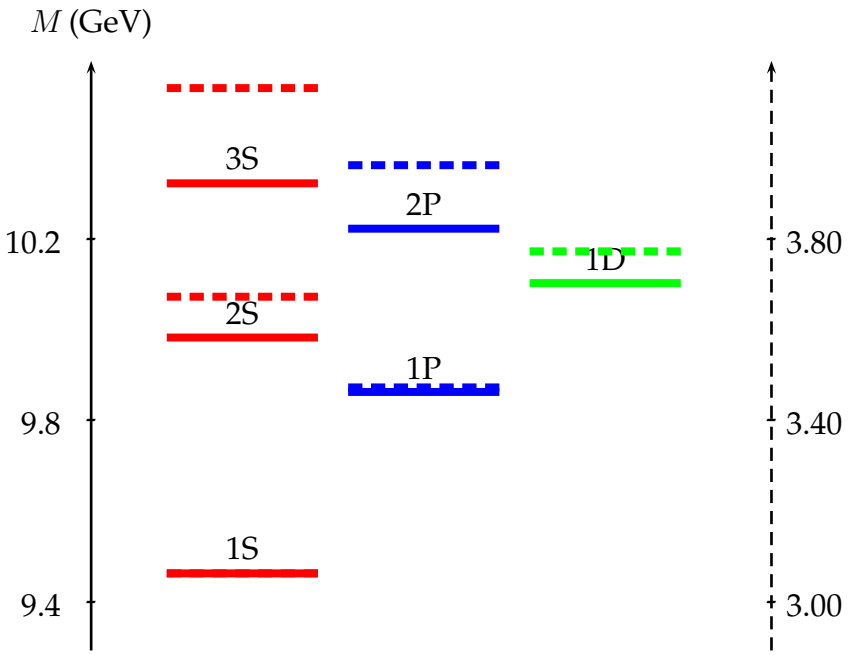

Figure 9: Comparison of the predictions for the $b\bar{b}$ (solid lines) and $(c\bar{c})$ (dotted lines) levels with a very simple potential.

3.4 Improvements to the potential

Perhaps, one should not try to improve too much the simple potential model of mesons. Take for instance proton–nucleus scattering in nuclear physics, where the so-called Glauber approximation works very well. For years, many corrections have been estimated, and each correction gave a large effect. At the end, one understood that the most important corrections cancel each other. There might be something similar in the quark model. Anyhow, let us list a few possible improvements.

3.4.1 More elaborate potentials

So far, we mentioned the simplest choices for the interquark potential, such as power-law, logarithm, or Coulomb-plus-linear.

The success of these empirical potentials, with a short-range part finite or less singular than $1/r$, and a confining part less sharp than r can be understood as follows: the simplest interaction corresponds to a linear confinement, with a cigar-shaped gluon flux linking the two gluons, and a Coulomb-part to which one-gluon exchange contributes. But the linear part is smoothed by pair-creation effects; if one tries to increase the separation between c and $\bar{c}$, a pair of light quark is created, and this results in a softening of the interaction [68, 69]. At short-distance, there is the asymptotic freedom, which weakens the Coulomb term, see, for instance, the potential by Buchmuller et al. [70] or Richardson [71].

3.4.2 Relativistic corrections

The simplest potential models use the Schrödinger equation. However, the kinetic energy turns out not to be very small as compared to the rest mass of the quarks. Models have been devised with a relativistic form of the kinetic energy

$$\frac{\boldsymbol{p}^2}{2\,m} \to \sqrt{m^2+\boldsymbol{p}^2}-m\,, \tag{14}$$

See, e.g., Basdevant and Boukraa [72], Godfrey and Isgur [73], etc. This is of course just one step towards a fully relativistic and covariant picture, with retardation effects, etc., one of the most serious attempts being the work of the Bonn group [74, 75], which later also described the baryons.

3.4.3 Strong decay of quarkonia

As already stressed, the miracle of charmonium and bottomonium is the suppression of the decay into light hadrons, which proceeds through internal $Q\bar{Q}$ annihilation into gluons and subsequent hadronisation of the emitted gluons. Hence the states below the relevant $(Q\bar{q})+(\bar{Q}q)$ threshold are narrow, and the states lying above easily decay into a pair of flavoured mesons, by a Zweig-allowed mechanism.

The most widely used model for describing $(Q\bar{Q}) \to (Q\bar{q}) + (\bar{Q}q)$ is based on quark pair creation. For, say

$$A(q_1, \bar{q}_2) \to B(q_1, \bar{q}_3) + C(q_3, \bar{q}_2) \; , \tag{15}$$

a prescription is given for the creation of the extra pair $(q_3, \bar{q}_3)$, and the amplitude includes the coefficients for the spin–isospin recoupling and the overlap of the wavefunctions, say

$$\mathcal{M} = \int \mathrm{d}\tau \Psi(1,\bar{2})\, \mathcal{O}(3,\bar{3})\, \Psi^*(1,\bar{3})\, \Psi(3,\bar{2}) \; , \tag{16}$$

where $\mathrm{d}\tau$ is meant for all relative variables. The most popular is the co-called $^3\mathrm{P}_0$ model proposed by Micu [76] and extensively used and developed by the Orsay group [77] and many others. The Cornell model of charmonium [78] is very similar.

The early applications were mostly for nodeless states, for which (16) gives some slight enhancement or suppression of $|\mathcal{M}|^2$ multiplying pure phase effects. The application to high charmonium states $\psi^{(n)}$ revealed more dramatic variations. In particular, if you consider the three OZI-allowed decays

$$\psi^{(n)} \to D\bar{D},\; D^*\bar{D} + \mathrm{c.c.},\; D^*\bar{D}^* \; , \tag{17}$$

each, schematically, call for a certain momentum p in the initial-state wave function. If p lies at a node or a bump, the transition is suppressed or enhanced. On the basis of simple spin counting, one expects for (17) that the rates, divided by phase-space, are proportional to

$$R[D\bar{D},\; D^*\bar{D} + \mathrm{c.c.},\; D^*\bar{D}^*] \propto 1:4:7 \; , \tag{18}$$

while the experimental results for $\psi(4.04)$ show a clear dominance of $D^*\bar{D}^*$. This led to suggest a molecular structure for this $\psi(4.04)$ [79] (see the section on multiquarks). In fact, the formalism of quark-pair creation shows that the decays into $D\bar{D}$ or $D^*\bar{D} + \mathrm{c.c.}$ are suppressed by the node structure of the initial state [80]. See also, [81, 82]. In a more detailed analysis, Eichten et al. [83] and Fernandez et al. [84] concluded that the situation is neither as simple a pure $(c\bar{c})$ with nodes or a pure molecule, namely that this state is a charmonium with an abundant molecular component.[5]

3.4.4 Coupling to decay channel

The coupling to $(Q\bar{q}) + (\bar{Q}q)$ also influence the energy and the internal structure of the quarkonia. The quark model is just a first approximation, with the minimal component in Fock space, i.e., the first term in

$$|\psi\rangle = a\,|c\bar{c}\rangle + \sum_i b_i\,|c\bar{c}q_i\bar{q}_i\rangle + \cdots \tag{19}$$

with the second term tentatively saturated by the lowest $D^{(*)}\bar{D}^{(*)}$ component and the D_s analogues. In the model by Eichten et al. [78, 82], these contributions are generated

[5] I thank F. Fernandez for an interesting correspondence on this point

from the main $c\bar{c}$ component by an explicit operator which creates a pair of light or strange quarks out of the vacuum. This provides the states above the threshold with an explicit decay width. The branching ratios are found to be in good agreement with the observed ones. There is also a dispersive part, i.e., a shift of the energy due to meson loops (see Fig. 10). If one introduces an explicit mass difference between D and D^*, these meson loops contribute to the fine and hyperfine splittings. See also [85], and the recent contributions by Barnes et al. [86], T. Burns [87], etc., where it is stressed that the loops with D, D^*, D_s, etc., tend to cancel one another.

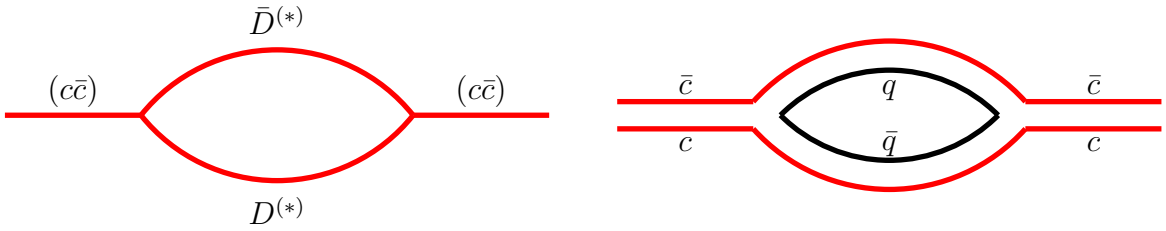

Figure 10: Mesonic loop correction to the charmonium, seen at the hadron level (left) or quark level (right)

However, very close to the $D\bar{D}$ threshold, there is an effect on the ψ', which can couple to $D\bar{D}$, while the η_c' only couples to $D\bar{D}^* + \text{c.c.}$ and $D^*\bar{D}^*$. Hence the $\psi' - \eta_c'$ is sensitive to the coupling to virtual decay channels [88,89].

We shall come back on these meson–meson configurations in the section on multi-quarks.

3.4.5 Fine structure

Three of the four members of the 1P level of $(c\bar{c})$ were discovered rather early, thanks to the transitions

$$\psi' \to \chi_J + \gamma\,, \quad \chi_J \to J/\psi + \gamma\,, \tag{20}$$

with $J = 0, 1, 2$. The masses have later been measured with a very high precision using an antiproton beam [90].

In the quark model, they are the spin triplet states $^3\mathrm{P}_0$, $^3\mathrm{P}_1$ and $^3\mathrm{P}_2$ (with a small admixture of $^3\mathrm{F}_2$). The formalism for the splittings and for the electromagnetic transitions is adapted from nuclear and atomic physics, and is reviewed, e.g., in Jackson's lectures at SLAC in 1976 [91].

The potential is written as

$$V(r) + V_{ss}(r)\,\boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2 + V_{ls}(r)\,\boldsymbol{\ell}.\boldsymbol{s} + V_t(r)\,S_{12}\,, \tag{21}$$

where $\boldsymbol{\ell}$ is the orbital momentum, $\boldsymbol{s} = (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)/2$ the total spin of the quarks, and the tensor operator is $S_{12} = 3\,\boldsymbol{\sigma}_1.\hat{r}\,\boldsymbol{\sigma}_1.\hat{r} - \boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2$. If the two last terms in (21) are treated at

first order, one gets the masses

$$\begin{aligned} M(^3\mathrm{P}_0) &= M_t - 2\,\langle V_{ls}\rangle - 4\,\langle V_t\rangle\ , \\ M(^3\mathrm{P}_1) &= M_t - \ \ \langle V_{ls}\rangle + 2\,\langle V_t\rangle\ , \\ M(^3\mathrm{P}_2) &= M_t + \ \ \langle V_{ls}\rangle - \frac{2}{5}\,\langle V_t\rangle\ , \end{aligned} \tag{22}$$

in terms of a an average triplet mass M_t and two matrix elements. This can be inverted as

$$\begin{aligned} M_t &= \frac{1}{9}\left[M(^3\mathrm{P}_0) + 3\,M(^3\mathrm{P}_1) + 5\,M(^3\mathrm{P}_2)\right]\ , \\ \langle V_{ls}\rangle &= \frac{1}{12}\left[-2\,M(^3\mathrm{P}_0) - 3\,M(^3\mathrm{P}_1) + 5\,M(^3\mathrm{P}_1)\right]\ , \\ \langle V_t\rangle &= \frac{5}{72}\left[2\,M(^3\mathrm{P}_0) - 3\,M(^3\mathrm{P}_1) + M(^3\mathrm{P}_1)\right]\ . \end{aligned} \tag{23}$$

Note that if the spin-orbit and tensor terms are treated beyond perturbation theory, leading to the computed masses $M(^3\mathrm{P}_J)$, and if M_t still denotes the computed result with only $V(r) + V_{ss}(r)$, then one gets

$$M_t \geq \frac{1}{9}\left[M(^3\mathrm{P}_0) + 3\,M(^3\mathrm{P}_1) + 5\,M(^3\mathrm{P}_2)\right]\ , \tag{24}$$

i.e., M_t is *above* the naive weighted average. This should be kept in mind, when estimating the sign of V_{ss} by comparing the spin-triplet states with the spin-singlet state governed by $V(r) - 3\,V_{ss}(r)$ (see below the paragraph on $^1\mathrm{P}_1$).

From the date of the Particle Data Group [92] on the 1P levels, of charmonium, (23) gives

$$M_t = 3.525\ , \quad \langle V_{ls}\rangle = 0.035\ , \quad \text{and} \quad \langle V_t\rangle = 0.01\,\mathrm{GeV}\ . \tag{25}$$

For the 1P level of $(b\bar{b})$, the result is

$$M_t = 9.900\ , \quad \langle V_{ls}\rangle = 0.014\ , \quad \text{and} \quad \langle V_t\rangle = 0.003\,\mathrm{GeV}\ , \tag{26}$$

and for 2P

$$M_t = 10.260\ , \quad \langle V_{ls}\rangle = 0.009\ , \quad \text{and} \quad \langle V_t\rangle = 0.002\,\mathrm{GeV}\ . \tag{27}$$

3.4.6 Hyperfine splittings

This is a more delicate issue on the experimental side. The simple mechanisms explaining the fine structure (see next section) also predicted some hyperfine splitting, with a pseudoscalar state named η_c about 0.1 GeV lighter than its vector partner J/ψ,. The η_c is the $^1\mathrm{S}_0$ state of $(c\bar{c})$, with $n = 1$. It was thus anticipated that the M1 transition from J/ψ to η_c would not have a very large probability [93, 94].

A first candidate for η_c was claimed in Germany [95], with a surprising shift $\delta = m(J/\psi) - m(\eta_c) \sim 0.3\,\mathrm{GeV}$. Leutwyler and Stern [96] claimed that such a large shift was

an unavoidable consequence of the relativistic character of the dynamics. In contrast, most potential builders predicted a smaller δ, once the first η_c was announced, recognized to face difficulties to accommodate such a large value of δ [97–101].

The German η_c was not confirmed [102], and instead, a more plausible one was seen at 2.980 GeV [103, 104], and confirmed in other places, including LEP and antiproton–proton collisions. The corresponding splitting is $\delta \simeq 0.116\,\mathrm{GeV}$.

The $\eta_c(2S)$, sometimes named η_c', was then predicted with a splitting $\delta' = \psi' - \eta_c'$ of about 80 MeV. The ratio δ'/δ is mainly governed by the ratio $R = |\phi_2(0)|^2/|\phi_1(0)|^2$ of the wave function at the origin. This ratio can be estimated in potential models (for instance, this ratio is exactly $R = 1$ for a purely linear potential). It can also be deduced from the ratio of the leptonic widths. The $\eta_c(2S)$ was unfortunately forgotten in the elaborate Cornell model with explicit account for the coupling to the virtual decay channels [82]. This omission was repaired in [88,89], and the effect of meson loops found to be sizeable. The $\eta_c(2S)$ does not couple to the nearby $D\bar{D}$, but the ψ' does, and is pushed down. This reduces δ'. The $\eta_c(2S)$ was eventually found at BELLE, BABAR and CLEO, in B decay, in $\gamma\gamma$ spectra and double-charmonium production [105–107]. It means that if one looks at $e^+e^- \to J/\psi + X$ and believes the Zweig rule, X is dominated by $(c\bar{c})$, and, indeed, already known states have been seen in X as striking peaks. The $\eta_c(2S)$ was confirmed in some other experiments. The splitting is $\psi' - \eta_c' = 0.049\,\mathrm{GeV}$.

From the very beginning, it was suspected that the h_c, corresponding to the $^1\mathrm{P}_1$ state of $(c\bar{c})$, will be very difficult to produce. See, e.g., Renard [108]. In most models, h_c is predicted to lie very close to the centre of gravity of the spin-triplet, M_t. A first indication was provided by the R704 experiment at CERN, the last experiment at the ISR ring [109]. The h_c was later found at CLEO (Cornell) [110] and E835 (Fermilab) [111]. The mass of h_c coincides with the centre of gravity of the triplet, and there is presumably a cancellation of several small terms.

The analogues in the $(b\bar{b})$ family were also hard to find. We have now η_b, $h_b(1P)$ and $h_b(2P)$ in the particle listings [92]. Very recently, CLEO [112] reported a measurement of $\eta_b(2S)$ and a confirmation of $\eta_b(1S)$. The results are

$$\delta(1S) = 0.067\,, \quad \delta(2S) = 0.049 \quad h_b(1P) = 9.898\,, \quad h_b(2P) = 10.260\,\mathrm{GeV}\,. \tag{28}$$

Two remarks are in order:

1. The value of $\delta(1S)$ for $(b\bar{b})$ was predicted by HPQCD [113] to be $70 \pm 9\,\mathrm{MeV}$. If you cannot afford lattice QCD, you can attempt a poor-man estimate from $(c\bar{c})$. Assuming a logarithmic potential, with the distances scaling as $m^{-1/2}$, and a hyperfine splitting proportional to $|\psi(0)^2|^2/m^2$, one gets [6]

$$\delta(1S)_{b\bar{b}} = \delta(1S)_{c\bar{c}} \left(\frac{m_b}{m_c}\right)^{-1/2} \sim 65\,\mathrm{MeV}\,. \tag{29}$$

[6] I remember doing this computation using the cell phone of Kamal Seth in a fancy restaurant of Munich

2. The ratio of $\delta(1S)$ to $\delta(2S)$ drops from about 2.3 to about 1.4 in $(c\bar{c})$ to $(b\bar{b})$. This reflects how anomalously high is the $\eta_c(2S)$ due to its vicinity of the OZI threshold.

3.4.7 The origin of the spin-dependent forces

The phenomenology of the spin-dependent forces in charmonium was greatly inspired by QED and by nuclear forces.

It was early stressed that the one-gluon-exchange contribution to the $c\bar{c}$ interaction is very similar to the Coulomb interaction in QED, and thus is associated to the same spin corrections. The name "chromomagnetism" was given to the analogue of the magnetic interaction. We shall come back on the important role of chromomagnetism in the systematics of hyperfine splittings in ordinary hadrons [114] and in speculations about possible multiquarks [115].

The one-meson-exchange picture of the nuclear forces, which was very popular in the 60s and early 70s. The nucleon–nucleon potential was written $V = \sum V_i$, where each V_i corresponds to a single partial wave in the t-channel, and thus gives a very specific spin dependence in the s-channel.

Schnitzer [97, 98], for instance, analysed the splittings assuming a vector meson exchange, and a scalar confinement. Thus the $1/r$ terms gives spin-spin, tensor and spin-orbit terms similar to the terms describing the fine and hyperfine structure of atoms, or similar to the terms associated with ω exchange in nuclear forces. The term proportional to r gives a negative spin-orbit force. Several variants were discussed by various authors [100, 116, 117].

The method to reach full consistency of this approach was explained by Gromes (see, e.g., [118] for references) and Eichten and Feinberg [119]. The problem is that the effective potential in the Schrödinger equation contains two types of contributions: an intrinsic spin dependence due to the nature of the exchanged object, and a relativistic effect known as Thomas precession. Even a scalar interaction, when reduced to an effective non-relativistic potential, contains a central term and a related spin-orbit one.

3.4.8 Orbital mixing

The states with natural parity, except $^3\mathrm{P}_0$, contains two partial waves, which can mix. Let us consider $J^{PC} = 1^{--}$. The wave–function reads

$$\psi = \frac{u(r)}{r}|^3\mathrm{S}_1\rangle + \frac{w(r)}{r}|^3\mathrm{D}_1\rangle \ , \tag{30}$$

and after some algebra, it can be shown that the coupled radial equations read

$$\begin{aligned} -\frac{u''(r)}{m} + V_c(r)\,u(r) + \sqrt{8}\,V_t(r)\,w(r) &= E\,u(r) \ , \\ -\frac{w''(r)}{m} + \left[\frac{6}{m\,r^2} + V_c(r) - 3\,V_{ls}(r) - 2\,V_t(r)\right] w(r) + \sqrt{8}\,V_t(r)\,u(r) &= E\,u(r) \end{aligned} \tag{31}$$

first derived many years ago for the deuteron. Here $V_C = V + V_{ss}$ is the spin-triplet central potential. Solving (31) requires a regularisation of $V_t(r)$ which behaves as r^{-3} at short distances in most models. Often the result, for e.g., ψ'' is written as [120]

$$\psi(3770) = a|^3\mathrm{D}_1, n = 1\rangle + b_1|^3\mathrm{S}_1, n = 1\rangle + b_2|^3\mathrm{S}_1, n = 2\rangle + \cdots \tag{32}$$

where the states in the r.h.s. are obtained by neglecting the coupling of the two equations. One could debate whether b_2 is larger than b_1 because ψ' lies very close to ψ'', or b_1 is larger due to the favourable node structure of J/ψ and ψ''. This is not obvious: in atomic physics, the analogue of (32) requires a summation over all b_i (in the discrete and continuous spectrum). See, e.g., [121, 122].

The orbital mixing is responsible for the coupling of $\psi(3770)$ to e^+e^- that made its observation possible. It might also influence the pattern of its hadronic decays. See, e.g., the discussion by Rosner [120].

Very likely, a good fraction of the orbital mixing is due to the coupling to decay channels. For instance, the coefficients in (32) are estimated in [89], which uses an improved version of the Cornell model.

3.5 Summary for heavy quarkonia

In Fig. 11, reproduced from [31] with the kind permission of the author, are shown the levels of charmonium. The states labelled X, Y, Z will be discussed in another section. It is remarkable to see exactly the levels predicted by the quark model. On the left, the spin singlet states, with relative angular momentum $\ell = 0$ for the two η_c, and $\ell = 1$ for h_c. In the middle, the various states with spin triplet, and $\ell = 0, 1$, and even one example with $\ell = 2$, and the various recoupling of spin and orbital momentum. The hierarchy of orbital vs. radial excitations, and the patterns of fine and hyperfine splittings are now well described with simple potentials that are now better understood from the underlying QCD.

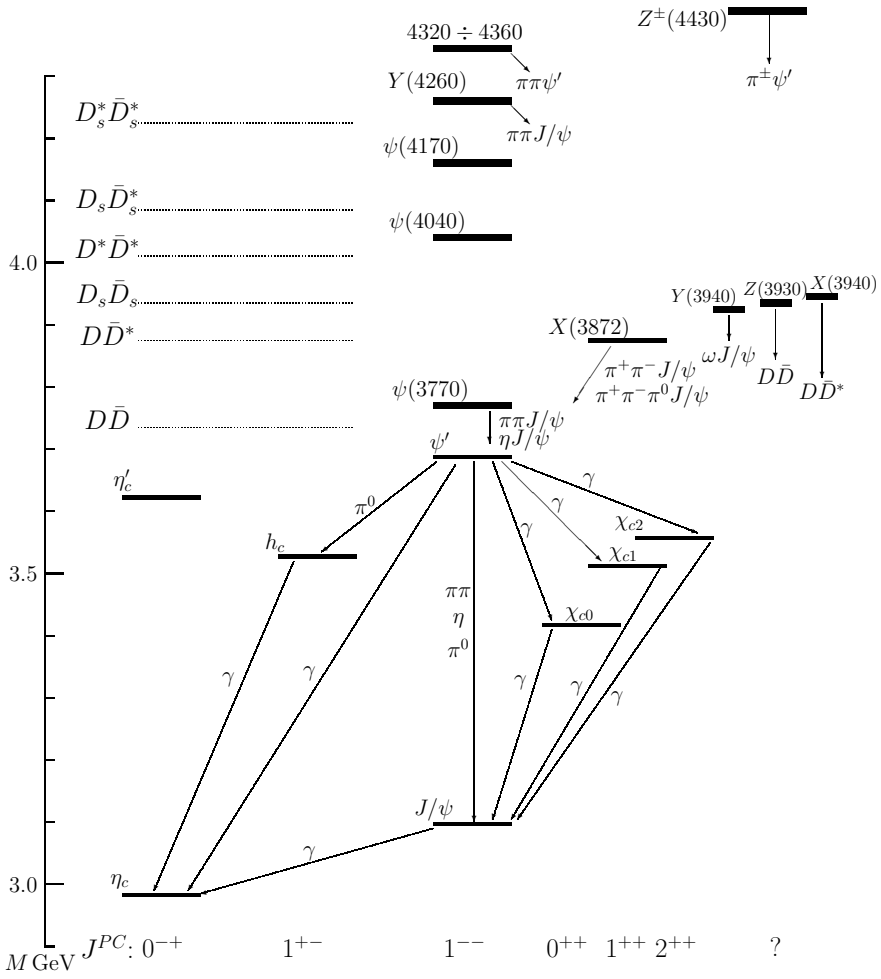

Figure 11: Levels of charmonium, and transitions among them (from [31]).

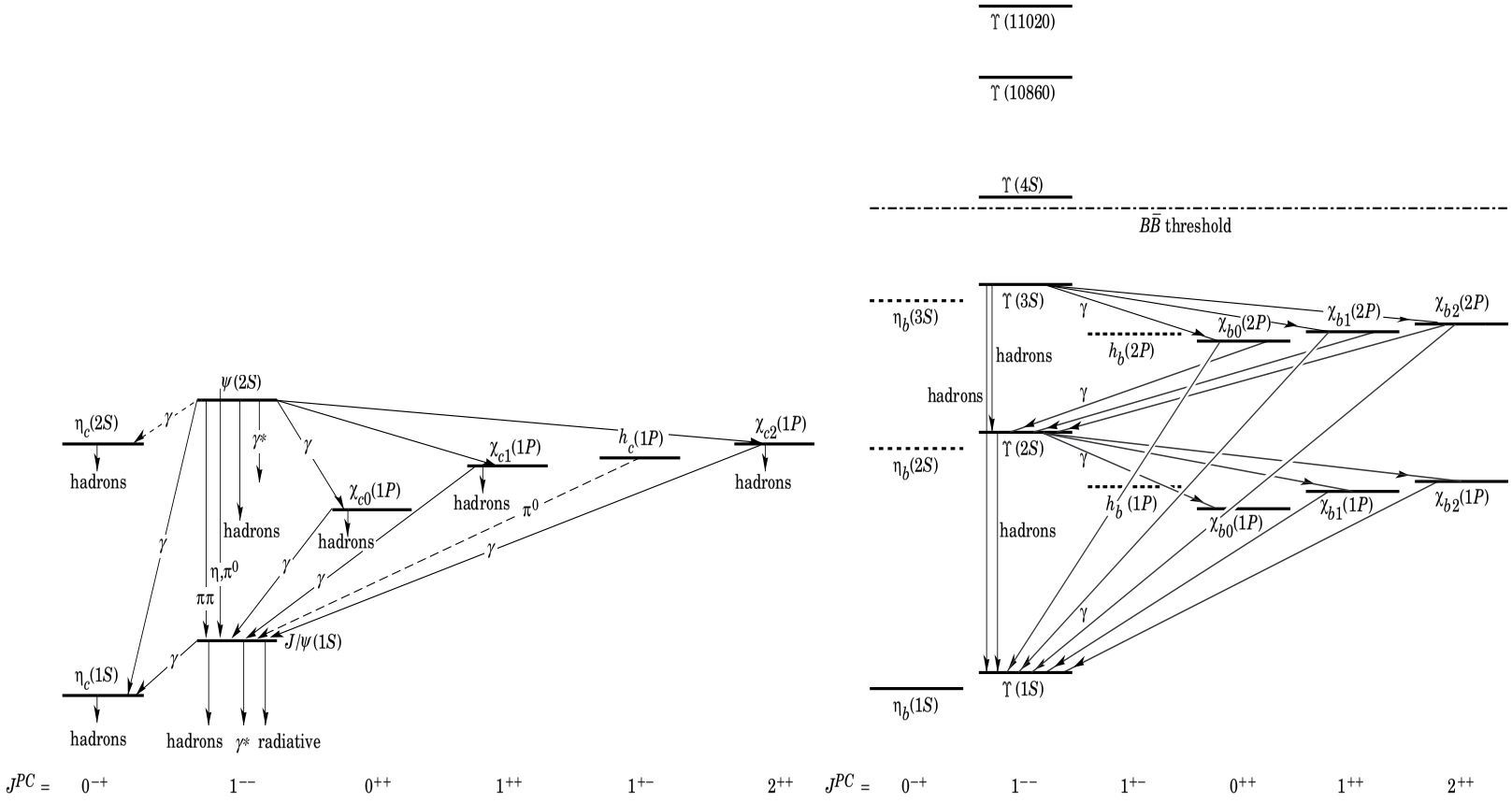

Figure 12: Charmonium and bottomonium spectra. Borrowed from PDG [92]. Some states indicated by a dotted line have been identified recently.

In Fig. 12, we reproduce the lower part of the charmonium spectrum, and the bottomonium spectrum, borrowed from the Review of Particle Properties [92].

The spectra are very similar. This is due to the property of flavour independence. The differences are also understood, due to explicit mass-dependence of the fine and hyperfine terms.

3.6 Light mesons

The simple quark model is in principle not applicable to light quarks, but some reckless attempts were encouraging. For instance, Martin [64] managed to fit $(b\bar{b})$, $(c\bar{c})$, $(s\bar{s})$ and $(c\bar{s})$ with a single potential. The game was extended further by Bhaduri et al. [123], however their uniform regularisation of the hyperfine interaction gave poor predictions for $J/\psi - \eta_c$ and similar splittings. Their model was refined by several authors, see, e.g., [124], with a regularisation of spin forces that depends on the system under consideration.

Isgur et al. [125] have explained why there are some good reasons to link potential models, with a minimal amount of relativity, to the underlying QCD.

Let us give two examples. The first positive-parity excitations of $(q\bar{q})$ with $I = 1$ are $a_0(980)$, $b_1(1235)$, $a_1(1260)$ and $a_2(1320)$. In the quark model, they correspond to the partial wave $^3\mathrm{P}_0$, $^1\mathrm{P}_1$, $^3\mathrm{P}_1$ and $^3\mathrm{P}_2$. The level order of this 1P multiplet, and even the pattern of spacings is very similar to what is observed in the charmonium 1P states.

If one looks at meson with high spin J and plot the square mass M^2 against J, one finds an almost perfect linear behaviour for this "Regge trajectory". If a linear potential is plugged into a Schrödinger equatiion with relativistic kinematics, this linear behaviour is reproduced.

3.7 Heavy-light mesons

This is an even more dangerous playing ground. Remember, for instance, that an electron is more relativistic in the hydrogen atom, (p, e^-), than in the positronium atom, (e^+, e^-). There is nevertheless a very interesting spectroscopy of D and B mesons.

In first approximation, the reduced mass governing the internal dynamics is the same for $(c\bar{q})$ and $(b\bar{q})$ mesons, so the excitation spectra should be very similar. This is rather obvious in potential models, but it remains valid much beyond this framework. This is one of aspects of "heavy quark symmetry" [126–128].

3.8 Mathematical developments

The quark model motivated some studies about the properties of the Schrödinger operator with confining interactions. See, e.g., [25,129] for a review and references.

Among the subjects, let us mention the following.

3.8.1 Level order

A sufficient condition has been derived that transforms the Coulomb degeneracy

$$E(1,\ell) = E(2,\ell-1) = \cdots = E(\ell+1,0)\ , \tag{33}$$

into a series of inequalities: the condition is related to the sign of the Laplacian of the potential. As $\Delta V > 0$ for the funnel potential (13) or any plausible interquark potential, this theorem explains the observed $1\mathrm{P} < 2\mathrm{S}$ ordering in quarkonium, and $2\mathrm{P} < 3\mathrm{S}$ in bottomonium. Note that $\Delta V \leq 0$ for the last electron of an alkalin atom and $\Delta V \geq 0$ for a muon inside a nucleus: this explains the observed breaking of the Coulomb degeneracy for alkalin atoms and muonic atoms.

Similarly, the sign of the second derivative of the potential as a function of r^2 governs the breaking of the HO degeneracy,

$$E(1,\ell) = E(2,\ell-2) = E(2,\ell-4) = \cdots\ . \tag{34}$$

This explains why $2\mathrm{S} < 1\mathrm{D}$, i.e., $m(\psi') < m(\psi'')$.

There are also results on the spacing of levels as a function of the radial number. For the harmonic oscillator, there is a remarkable equal spacing, $E(n+1,\ell) - E(n,\ell) = E(n,\ell) - E(n-1,\ell)$. For a linear potential, the spacing decreases.

3.8.2 The wave function at the origin

Many decay widths are proportional to the square wave function at the origin, which for S-states, reads

$$p_n = |\Phi_n(0)|^2 = \frac{1}{4\,\pi} u_n'(0)^2\ . \tag{35}$$

if the reduced radial wave function is arranged to be real. The studies on quarkonium gave the opportunity to remember the rule by Schwinger (see, e;g., [129])

$$u'(0)^2 = 2\,\mu \int_0^\infty \frac{\mathrm{d}V}{\mathrm{d}r} u^2(r)\,\mathrm{d}r\ . \tag{36}$$

This explains why the p_n are independent of n for a purely linear interaction, and, conversely, why the normalisation factor in (12) is given by the derivative.

There are sufficient conditions ensuring that p_n increases or decreases as a function of n [25].

3.8.3 The dependence upon the masses

If flavour independence is accepted as a strict rule, one has to study how the energy levels and the wave functions evolve when the reduced mass is modified.

As the kinetic energy term is $\boldsymbol{p}^2/(2\,\mu)$, obviously $E \searrow$ as $\mu \nearrow$. The effect is pronounced when going from $(c\bar{c})$ to $(b\bar{b})$. On the other hand, the reduced mass governing

$(Q\bar{q})$ does not change much from $Q = c$ to $Q = b$. Hence the OZI threshold $(Q\bar{q}) + (\bar{Q}q)$ has almost the same energy for $Q = b$ and $Q = c$. This explains why the $\Upsilon(1S)$ is more deeply bound than J/ψ with respect to this threshold, and why there are more narrow states in the bottomonium spectrum than in the charmonium one.

One can go further and notice that in a strictly flavour-independent interaction, the inverse reduced mass enters linearly. There is a known result that for an Hamiltonian depending linearly on a parameter, say $H = A + \lambda B$, the ground state energy ϵ_0, or the sum of first energies $\sum_{i=0}^{N} \epsilon_i$ is a concave (or convex upward) function of λ. Once the constituent masses are added, this implies for the ground state of each angular-momentum sector,

$$(Q\bar{Q}) + (q\bar{q}) \leq 2(Q\bar{q}) \; , \tag{37}$$

an inequality independently derived by Martin and Bertlmann [25, 130] and Nussinov [131, 132] for potential models, and extended much beyond this framework [132, 133]. Early applications introduced dangerously light quarks, but still, the inequality is satisfied for the spin-triplet states of $(c\bar{c})$, $(s\bar{s})$ and $(c\bar{s})$. Safer is the prediction that the (spin averaged) ground state of $(b\bar{c})$ is above the middle of $(c\bar{c})$ and $(b\bar{b})$.

Note that in atomic physics, it is reasonable to believe that the Born–Oppenheimer potential that binds di-atomic molecules remains unchanged if the nuclei are replaced by isotopes. Thus (37) holds for H_2, D_2 and HD, and this governs their abundance at thermodynamic equilibrium [134].

3.8.4 Concavity with respect to the spin coefficients

In the same vein, one can explain the sign of the error when treating the spin corrections to first order. For the hyperfine effect of S-state alone the singlet and triplet Hamiltonians are respectively, in an obvious notation,

$$H_s = H_0 - 3\, V_{ss} \; , \quad H_t = H_0 + V_{ss} \; , \tag{38}$$

so that the fictitious eigenstate of H_0 alone lies at

$$E_0 \geq \frac{1}{4} E_s + \frac{3}{4} E_t \; . \tag{39}$$

Note that the inclusion of the tensor force would slightly lower E_t, with $^3\mathrm{S}_1$ becoming $^3\mathrm{S}_1 - {}^3\mathrm{D}_1$.

It is straightforward to extend this to the case of two parameters. Consider the generic spin-triplet Hamiltonian

$$H_J = H_t + \alpha_J\, V_{ls} + \beta_J\, V_t \; , \tag{40}$$

so that $H_0 + 3\, H_1 + 5\, H_2 = 9\, H_t$, then the fictitious spin-triplet state free of spin-orbit and tensor corrections lies at

$$E_t \geq \left[E_{J=0} + 3\, E_{J=1} + 5\, E_{J=2} \right] / 9 \; , \tag{41}$$

i.e., above the naive centre of gravity, as discussed earlier in this section. Thus if the $^1P_1$ is found at the location of the naive of gravity, it has received some *attractive* spin–spin downward shift.[7]

4 Baryons

Tres faciunt collegium[8]
Latin sentence

4.1 Introduction

The quark model of baryons was developed in the 60s, in particular by Dalitz and his collaborators, and several other groups, at a time when many nucleon and hyperon resonances were already known.

These early studies, and the more recent work by Isgur and Karl, Gromes et al., Stancu et al., Cutosky et al., cannot be separated from the harmonic oscillator (HO) model, which provides a powerful classification scheme, and an efficient tool to implement the symmetry constraints. This is reviewed, e.g., by Hey and Kelly [135].

The HO model has reached a high degree of refinement, with all the possible corrections carefully listed, and their tentative interpretation in QCD (one-gluon-exchange).

There has also been attempts to solve the three-body problem, using known techniques developed in atomic or nuclear physics, such as Faddeev equations, hyperspherical harmonics or variational methods. Then any potential can be used, for instance a funnel interaction with a superposition of Coulomb and linear contributions.

Note that in most early studies, it was implicitly assumed that the potential is pairwise, and the question has been addressed of the link between the interquark potential in baryons and the quark–antiquark potential in mesons. This will be discussed in Sec. 4.12. In fact, it has been anticipated for many years, that the confining interaction is of three-body nature, i.e., depends simultaneously upon all relative distances. This makes the calculations technically more involved, but makes a link with the lattice QCD which favours such potentials [136].

The role of relativistic effects is of course important. In the latest works by Isgur et al., a relativistic kinematics was adopted [137]. There are even models based on the Bethe–Salpeter equation [138].

4.2 Jacobi coordinates

In the case of equal masses, a set of Jacobi coordinates that diagonalise the kinetic energy is (see Fig. 13)

$$\boldsymbol{R} = \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3}{3} \ , \quad \boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1 \ , \quad \boldsymbol{y} = \frac{2\,\boldsymbol{r}_3 - \boldsymbol{r}_1 - \boldsymbol{r}_2}{\sqrt{3}} \ , \tag{42}$$

[7] I remember discussions on this point at a Workshop organised in Genoa, to celebrate years of charm physics with antiprotons, and later with Kamal Seth.

[8] Three makes a company

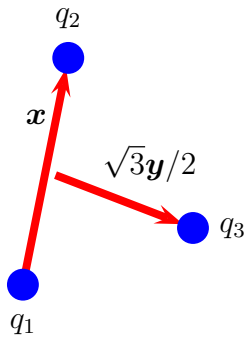

Figure 13: Jacobi coordinates for the relative motion of the quarks inside a baryon

where the factor in $\boldsymbol{y}$ makes it easier to implement the symmetry constraints, as explained shortly. Then, once the centre of mass motion is removed, the intrinsic Hamiltonian reads

$$h = \frac{\boldsymbol{p}_x^2}{m} + \frac{\boldsymbol{p}_y^2}{m} + V(\boldsymbol{x}, \boldsymbol{y}) \,, \tag{43}$$

where the potential energy, already written as translation-invariant, has to be scalar.

In the case of two different masses, say (m, m, M), $\boldsymbol{R}$ is modified, but one can keep $\boldsymbol{x}$ and $\boldsymbol{y}$ as above. Then the intrinsic Hamiltonian becomes

$$h = \frac{\boldsymbol{p}_x^2}{\mu_x} + \frac{\boldsymbol{p}_y^2}{\mu_y} + V(\boldsymbol{x}, \boldsymbol{y}) \,, \tag{44}$$

with the reduced masses now given by

$$\mu_x = m \,, \quad \mu_y^{-1} = (m^{-1} + 2\, M^{-1})/3 \,. \tag{45}$$

If the three masses are unequal, the Jacobi coordinate $\boldsymbol{y}$ becomes $\boldsymbol{y} \propto (m_1 + m_2)\boldsymbol{r}_3 - m_1\, \boldsymbol{r}_1 - m_2\, \boldsymbol{r}_2$, and it is straightforward to derive the reduced masses μ_x and μ_y as functions of the individual masses m_i.

Later in this section, we shall elaborate more on the potential energy V. Let us just mention here two popular choices. The harmonic oscillator corresponds to $V = K(r_{12}^2 + r_{23}^2 + r_{31}^2)$, where $r_{ij} = |\boldsymbol{r}_{ij}|$ and $\boldsymbol{r}_{ij} = \boldsymbol{r}_j - \boldsymbol{r}_i$. It becomes $V = 3\, K(\boldsymbol{x}^2 + \boldsymbol{y}^2)/2$ for (m, m, m) and (m, m, M), and a more complicated quadratic form of $\boldsymbol{x}$ and $\boldsymbol{y}$ for (m_1, m_2, m_3). The pairwise models reads $v(r_{12}) + v(r_{23}) + v(r_{31})$.

4.3 Permutation symmetry

Schematically, the wave function of a baryon in the quark model is

$$\Psi = \psi(\boldsymbol{x}, \boldsymbol{y})\, \psi_s\, \psi_i\, \psi_c \,, \tag{46}$$

with orbital, spin, isospin and colour parts, and this wave function has to be antisymmetric with respect to the permutation of the identical quarks.

If there are two identical quarks, as in $\Xi^- = (ssd)$ or in $\Lambda = (uds)$ in the limit where isospin symmetry is exact, then each term in (46) is either symmetric or antisymmetric. For the ground state of Λ, with isospin $I = 0$, the colour, spin and isospin parts are antisymmetric and the orbital part symmetric. This means spin 0 for the light quarks, and thus $s = 1/2$ for the three quarks, isospin $I = 0$, and $\psi(\boldsymbol{x}, \boldsymbol{y})$ even in $\boldsymbol{x}$. For instance, in the harmonic oscillator, $\psi(\boldsymbol{x}, \boldsymbol{y}) \propto \exp[-a\,\boldsymbol{x}^2 - b\,\boldsymbol{y}^2]$.

There are two type of P-states: one where the degree of freedom associated to $\boldsymbol{\lambda}$ is excited, with the same symmetry pattern as the ground state. Another wave where the light quarks have their spin coupled to $s_{qq} = 1$, and the orbital wave function $\psi(\boldsymbol{x}, \boldsymbol{y})$ is odd in $\boldsymbol{x}$. In the harmonic oscillator, the orbital wave functions are of the type

$$\boldsymbol{y}\, \exp[-a\,\boldsymbol{x}^2 - b\,\boldsymbol{y}^2]\,, \qquad \boldsymbol{x}\, \exp[-a\,\boldsymbol{x}^2 - b\,\boldsymbol{y}^2]\,. \tag{47}$$

For the former, the quark spin $\mathsf{s} = 1/2$ and the angular momentum $\boldsymbol{\ell}_y$ gives a baryon spin $J = 1/2$ or $J = 3/2$. For the latter, both $\mathsf{s} = 1/2$ and $\mathsf{s} = 3/2$ are possible, and thus a variety of values for the spin J. Usually, the spin orbit splittings are small in baryons (see below), and the states with various combinations of spins and angular momenta are nearly degenerate. One exception is the $\Lambda(1405) - \Lambda(1520)$ pair, which has motivated an abundant literature. The most plausible explanation is the coupling of one of these states to a nearby threshold.

For three identical quarks, imposing the constraints of antisymmetrisation is more delicate. The colour coupling $3 \times 3 \times 3 \to 1$ is antisymmetric, thus the rest of the wave function should be symmetric. This was, indeed, one of the motivations for introducing colour. For the Ω^-, this is rather simple: the spin wave-function is symmetric, isospin does not exists, and the orbital wave function is symmetric. For instance, $\psi(\boldsymbol{x}, \boldsymbol{y}) = \exp[-a(\boldsymbol{x}^2 + \boldsymbol{y}^2)]$ in the HO case. This is similar for the $\Delta(1232)$ multiplet, as the isospin coupling $1/2 \times 1/2 \times 1/2 \to 3/2$ is symmetric.

For the first parity excitations of Ω^-, with $J^P = (1/2)^-$ or $(3/2)^-$, the spin and the orbital wave functions are neither symmetric nor antisymmetric, but their combination is symmetric. One cannot escape here to introduce the concept of *mixed symmetry*. The prototype is given by the Jacobi coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$ of (42). The permutation P_{12} reveals $\boldsymbol{x}$ odd and $\boldsymbol{y}$ even, but a circular permutation $P_\to$ gives

$$P_\to[\boldsymbol{y} + i\,\boldsymbol{x}] = j\,[\boldsymbol{y} + i\,\boldsymbol{x}]\,, \tag{48}$$

where $j = \exp(2\,i\,\pi/3)$, as usual. Obviously, if two pairs, say $z = v + i\,u$ and $Z = V + i\,U$, have the same permutation properties as $\boldsymbol{z} = \boldsymbol{y} + i\,\boldsymbol{x}$, then their coupling give a symmetric term, and antisymmetric one, and a new pair of mixed symmetry (again in the complex notation), respectively

$$\begin{aligned} \Re\mathrm{e}[z\,Z^*] &= u\,U + v\,V\,, \\ \Im\mathrm{m}[z\,Z^*] &= v\,U - u\,V\,, \\ [z\,Z]^* &= (u\,U - v\,V) - i\,(u\,V + v\,U)\,. \end{aligned} \tag{49}$$

In the case of the P-states of Ω^-, we have a pair of mixed-symmetry orbital wave functions, $\{\psi_x, \psi_y\}$, with either $\boldsymbol{x}$ or $\boldsymbol{y}$ excited, which for the harmonic oscillator read $\{\psi_x, \psi_y\} = \{\boldsymbol{x}, \boldsymbol{y}\}\, \exp[-a(\boldsymbol{x}^2 + \boldsymbol{y}^2)]$, and a pair of spin 1/2 wave functions

$$S_x = \frac{1}{\sqrt{2}}\,[\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow]\ , \quad S_y = \frac{1}{\sqrt{6}}\,[2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow]\ , \tag{50}$$

which bears an obvious analogy with the Jacobi variables (42). With a spin-independent interaction, the P-states of Ω^- have a wave function of the type $[\psi_x\, S_x + \psi_y\, S_y]/\sqrt{2}$.

For the nucleon ground state, the orbital part is symmetric, but the spin and isospin parts are both of mixed symmetry, with (50) for the spin, its analogues $\{I_x, I_y\}$ for isospin, and the combination $[S_x\, I_x + S_y\, I_y]/\sqrt{2}$ being symmetric.

The most interesting example of baryon in this respect consists of the antisymmetric spin–isospin wave function $[S_x\, I_y - S_y\, I_x]/\sqrt{2}$ which requires a fully antisymmetric orbital wave function. This wave function is excited in any interquark separation. For instance, in the specific case of the harmonic oscillator, it reads $\boldsymbol{x} \times \boldsymbol{y}\, \exp[-a(\boldsymbol{x}^2 + \boldsymbol{y}^2)]$, i.e., $\ell^P = 1^+$. This is one of the states predicted in the three-quark model and absent in the quark-diquark model. See below the section on diquarks.

4.4 Solving the three-body problem for baryons

4.4.1 The harmonic oscillator

After rescaling, the linear oscillator reads $h = p^2 + x^2$ with energies $\epsilon_n = 1 + 2\,n$ and wave functions $\varphi_0(x) \propto \exp(-x^2/2)$ and $\varphi_n(x) \propto [-\mathrm{d}/\mathrm{d}x + x]^n\, \varphi_0(x)$.

For equal masses m, the three-body problem with $V = 2\,K/3(r_{12}^2 + r_{23}^2 + r_{31}^2)$ is governed by the Hamiltonian

$$H(m,m,m) = \frac{\boldsymbol{p}_x^2}{m} + \frac{\boldsymbol{p}_y^2}{m} + K(\boldsymbol{x}^2 + \boldsymbol{y}^2)\ , \tag{51}$$

which corresponds to a sum of two independent spatial oscillator, or six linear oscillators. The energies $E = (6 + 4\,n_x + 2\ell_x + 4\,n_y + 2\,\ell_y)\sqrt{K/m}$ have a cumulated number of excitations $N = 2\,n_x + \ell_x + 2\,n_y + \ell_y$. Here, the radial and orbital excitations in each variable are counted separately.

There is a considerable degeneracy of the levels of given $N \geq 2$, which contains contains several J^P states. The literature on baryons within HO has developed a systematic notation which is used beyond this model. In particular, the multiplets are denoted $[d, \ell^P]^{(n)}$, where d is the spin–flavour multiplicity, ℓ the total orbital momentum, and (n) the radial occurrence ($n = 0$ is omitted). For instance, the ground state is labelled as $[56, 0^+]$, and the Roper resonance and its partners $[56, 0]'$. The ground state contains the octet of Fig. 1 and the decuplet of Fig. 2, i.e., $2 \times 8 + 4 \times 10 = 56$ states. The first orbital excitation is $[70, 1^-]$, and the first state with a full antisymmetric orbital wave function is $[20, 1^+]$.

The case of two different masses reads

$$H(m,m,M) = \frac{\boldsymbol{p}_x^2}{m} + \frac{\boldsymbol{p}_y^2}{\mu} + K(\boldsymbol{x}^2 + \boldsymbol{y}^2) \; , \tag{52}$$

with μ given by (45). There is still an exact decoupling, and the energies are

$$E(m,m,M) = \sqrt{\frac{K}{m}}(3 + 4\,n_x + 2\,\ell_x) + \sqrt{\frac{K}{\mu}}(3 + 4\,n_y + 2\,\ell_y) \; . \tag{53}$$

Here the lowest excitation is explicitly associated with the heavier quark(s),i.e., this is an excitation in the ${}_v ecx$ variable. This point will be further discussed in the section on double-charm baryons.

The case of three different masses is also solvable exactly, though this is slightly less known.[9] Once the kinetic energy is written as (44), one can use a rescaling $\boldsymbol{x} \to \boldsymbol{x}/\sqrt{\mu_x}$ and $\boldsymbol{y} \to \boldsymbol{y}/\sqrt{\mu_y}$. The Hamiltonian becomes

$$H(m_1,m_2,m_3) = \boldsymbol{p}_x^2 + \boldsymbol{p}_y^2 + A\,\boldsymbol{x}^2 + B\,\boldsymbol{y}^2 + 2\,C\,\boldsymbol{x}\,\boldsymbol{y} \; , \tag{54}$$

and one is left with the diagonalisation of a 2×2 matrix.

4.4.2 First-order perturbation around the harmonic oscillator

We now turn to more general models for the interaction. The most spread method consists of writing the potential as

$$V = K(\boldsymbol{x}^2 + \boldsymbol{y}^2) + \delta V \; , \tag{55}$$

and treat the second term as a perturbation. If K is optimised, this is the first step of a variational expansion, to be discussed below. This is of course not very accurate, especially at short distances. If δV tentatively pushes down the Roper resonance (to be discussed shortly) from the $N = 2$ level of the harmonic oscillator to the vicinity of the $N = 1$ level, then perturbation theory is in principle not applicable, as it requires that the shifts are small compared to the initial spacings.

Isgur and Karl [139] have noticed the nice patterns of the $N = 2$ multiplet splitting when perturbed by a pairwise δV. It is illustrated in Fig. 14. Gromes and Stamatescu [140] pointed out that the upper part survives if the HO is perturbed by a symmetric 3-body potential, while the $[56, 0^+]'$ decouples. The same conclusion was reached by Taxil et al. [141], in the more general situation of a nearly hypercentral potential.

The upper part of Fig. 14 fits rather well the spin-averaged values of the observations.

This pattern and the one for the $N = 3$ multiplet have been further discussed by Taxil et al. [141], Stancu et al. [142], etc.

[9] I remember discussions on this point with J.-L. Basdevant, and with D.O. Riska

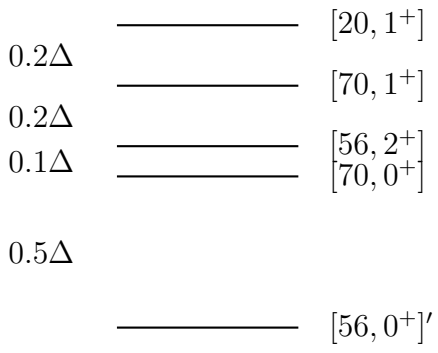

Figure 14: Splitting pattern of the $N = 2$ multiplet for a two-body perturbation of the harmonic oscillator. [Check whether the 2-body character is required.] There is an overall shift (not shown), and, in addition, a shift 0.1Δ, 0.2Δ, 0.3Δ or $\pm 0.5\Delta$ of the sublevels.

4.4.3 Converged variational methods

Let us consider the ground state with equal masses, but the generalisation to unequal masses is straightforward. The above method can be read as the first term in a systematic expansion

$$\Psi(\boldsymbol{x}, \boldsymbol{y}) = \sum_n c_n \, \phi_n(\boldsymbol{x}, \boldsymbol{y}) \; , \tag{56}$$

where n denotes the number of excitations and other quantum numbers associated to the solutions $\phi_n(\boldsymbol{x}, \boldsymbol{y})$ of the harmonic oscillator. The matrix elements can be calculated with subtle recursion relations. The convergence as a function of the maximal n is excellent for the energy but elusive for the short-range correlations.

A variant is the so-called Gaussian expansion, widely used in theoretical chemistry and other quantum problems. It reads

$$\Psi(\boldsymbol{x}, \boldsymbol{y}) = \sum_i \gamma_i \, \exp[-(a_i \, \boldsymbol{x}^2 + b_i \boldsymbol{y}^2 + 2 \, c_i \, \boldsymbol{x}.\boldsymbol{y})] \; . \tag{57}$$

Note that the permutation symmetry is violated in an individual term if $a_i \neq b_i$ and $c_i \neq 0$, but is restored in the summation either explicitly (correlated Gaussians) or by the numerical adjustment. Different techniques are used [143, 144] to avoid redundancies in the expansion and guide towards an efficient optimisation.

4.4.4 Faddeev equations

The Faddeev equations for the three-body problem have been written for short-range potentials in momentum space. Later, they have been translated into position space

and adapted to the confining interactions. They imply a summation over the internal angular momentum, but even with the crudest truncation, they provide an excellent wave function. The first paper (to my knowledge) where the Faddeev equations are applied to baryons is [145]. See, also, [146].

4.4.5 Hyperspherical expansion

The 6-dimensional vector $\{\boldsymbol{x}, \boldsymbol{y}\}$, car be written (ϱ, Ω_5) in spherical coordinates. The five angles Ω_5 include the physical angles $\hat{x}$ and $\hat{y}$ and $\arctan(y/x)$. The method consists of expanding $\Psi(\boldsymbol{x}, \boldsymbol{y})$ into partial waves in this space. For a harmonic oscillator, $V \propto \varrho^2$, and one recovers the familiar results. Otherwise, one gets an infinite set of coupled equations

$$-u''_{[L]}(\varrho) + \frac{(L+3/2)(L+5/2)}{\varrho^2}\, u_{[L]} + \sum_{[L]'} V_{[L],[L]'}(\varrho)\, u_{[L]'}(\varrho) = E\, u_{[L]}(\varrho)\ , \qquad (58)$$

where $[L]$ denotes the "grand" angular momentum and its associated magnetic numbers.

Keeping only one term in the expansion, the lowest hyperspherical harmonics, gives the hyperscalar approximation, which is already very good, as often stressed by the Genoa group [147]. The first application of this method to baryons is by Guimaraes et al. [148], and Hasenfratz et al. [149]. The convergence was studied in the thesis work by Taxil, and is summarised in [146, 150].

The splitting pattern of Fig. 14 has been re-analysed, with the starting point being now the hyperscalar approximation. The upper part is not modified, but the radial excitation now decouples. This gives more freedom to push this state down, but this is not sufficient. Høgaasen and I have noticed that the same hyperspherical potential V_{00} governs the ground state, its radial excitations and the first negative parity excitation. The effective angular momentum in the radial equation is $\ell = 3/2$ for the former, and $\ell = 5/2$ for the latter. Then for any reasonable model for the interquark interaction, the radial excitation is *above* the orbital one [151]. The result relies on the local character of the interaction and remains valid for local three-body terms. It does not hold if the potential is velocity or spin dependent.[10]

4.5 Light baryons

This is the first sector to which the quark model was applied. One cannot compare the refinement of the model and the quality of the fit with the analogues for heavy quarkonia. Nevertheless, the quark model explains the rough features of the observed spectrum, in the sense that any low-lying resonance corresponds to a level predicted in the quark model.

[10] I thank W. Plessas for an enlightening discussion on this point, many years ago.

Among the problem is that of the Roper resonance. This denotes the state $N^*(1440)$ with the same $I = 1/2$ and $J^P = (1/2)^+$ as the nucleon. Experimentally it lies at about the same mass as the orbital excitations of negative parity, perhaps slightly below. In the simplest quark model, it is a radial excitation. In the harmonic oscillator, it belongs to the $N = 2$ multiplet and the first orbital excitation to $N = 1$. With a standard Coulomb + linear interaction, with an accurate treatment of the three body problem, there is no way to make these states degenerate. This is a serious limitation for the naive quark model, which misses the relativistic effects and chiral dynamics. For instance a model with Goldstone-boson exchange, proposed by Glozman and Riska [152], and further developed by the Graz group [153, 154], gets the Roper resonance right. See, also, [155], and the comments by Isgur on this approach [156]. In recent lattice simulations with large pion mass (this facilitates the computation, see next lectures), the same ordering as potential models is obtained. When the pion mass decreases (and the cost of the computations increases), there is a dramatic change in the level ordering, and the experimental one is recovered [157].

4.6 Missing states and the diquark alternative

Conversely, there are several instances where a state predicted by the three-quark model is missing in the experimental spectrum. As often stressed by Isgur and Karl (their papers in the bibliography and private communications), most experimental data deal with pion or photon scattering off a nucleon. This favours states with one quark excited and the two remaining quarks nearly untouched, and suppresses states with double internal excitation. However, experiments with higher statistics should be able to see the missing states.

A drastic alternative is the diquark models. I should better say the diquark models, as there exists different points of view, and also, the model is regularly reinvented by authors who do not quote the pioneering papers. For a rather comprehensive review, see [158].

Diquarks have been used for many purposes, in particular multiparticle production. In spectroscopy, the quark–antiquark model retains only those baryons consisting of a compact diquark and a quark. The question is whether the diquark is postulated as an elementary constituent or somehow generated dynamically.

Martin has given a partial answer [159]: for three-quark systems with high orbital momentum ℓ, the ground state consists of two quarks mainly in S-wave with the third quark carrying most of the orbital momentum, while for low ℓ, the three quarks share ℓ symmetrically. This explained, at last, a long standing issue: the Regge trajectories of mesons and baryons are linear and have the same slope.

A digression here about the Regge trajectories. They were invented to explains some properties of potential scattering, and later, in a different form, for the high-energy scattering of hadrons. In this later context, a Regge trajectory is the squared mass of hadrons, as a function of the spin J. The observed trajectories are beautifully linear.

For mesons, this was interpreted a flux of constant section linking the quark to the antiquark.

An intriguing observation is that the baryons also belong to linear trajectories, with the same slope. This is of course natural in the quark–diquark model, as the energy is that of colour 3 linked to a colour $\bar{3}$. We now understand that this occurs spontaneously.

A question that is sometimes eluded is whether diquark are just an effective degree of freedom that simplifies the dynamics of baryons, or a constituent to be used on the same footing as quarks in the "lego" puzzle of hadrons. Recently, Maiani and his collaborators described the new X, Y, Z resonances as diquark–antidiquark states [160], as done earlier for baryonium. Frederikkson et al. [161] raised the important question for the three-diquark systems (though there is a slight confusion with the quantum numbers, the paper remains relevant): do we have a "demon" deuteron made of three diquark, in addition to the ordinary deuteron made of two baryons?

4.7 Baryons with a single heavy flavour

Some years ago, this sector was marginal for quark dynamics, with only the ground state of $(Quu), (Qud), \ldots, (Qss)$. Of course, the measurement of the lifetimes and semileptonic branching ratios were very exciting, to compare the role of W-emission, W-exchange, W-formation in various flavour configurations. But this is out of the scope of this review.

In recent years, a lot of data came from CLEO, B-factories, Tevatron, etc., and we expect many results from LHCb. It has even been suggested that we can learn more on light quark dynamics for the (Qqq) systems than from (qqq). This is a domain where the data on charm are or will be confirmed by the data on beauty.

Just to give an example, take Ω_b. When its mass was published by D0 [162], it was a surprise that $\Omega_b - \Lambda_b$ was found significantly higher than $\Omega_c - \Lambda_c$. See, e.g., [28, 163]. The CDF collaboration got a lower mass [164], as this is now confirmed by LHCb [165]. The mass of this state was predicted by Lipkin et al. [166][11], on the basis of the chromomagnetic interaction [114, 115]. See, also, [167, 168]. The lesson is that the spectrum has regularities, due in particular to flavour independence, and any departure should receive an explanation or disappear.

The spectrum of single-charm and single-beauty baryons are shown in Fig. 15. It includes the very recent discoveries at LHC [169, 170]. The spectra are arranged so that Λ_c and Λ_b coincide. Clearly the excitation energies are very similar.

4.8 Baryons with two heavy quarks

It becomes intriguing or even annoying that no (QQq) has been firmly identified yet. We have just the results by the SELEX collaboration at Fermilab [171, 172], and negative results in a few other attempts. As Brodsky often insists in Conference talsk or private

[11] I thank Marek Karliner for a correspondence on this point.

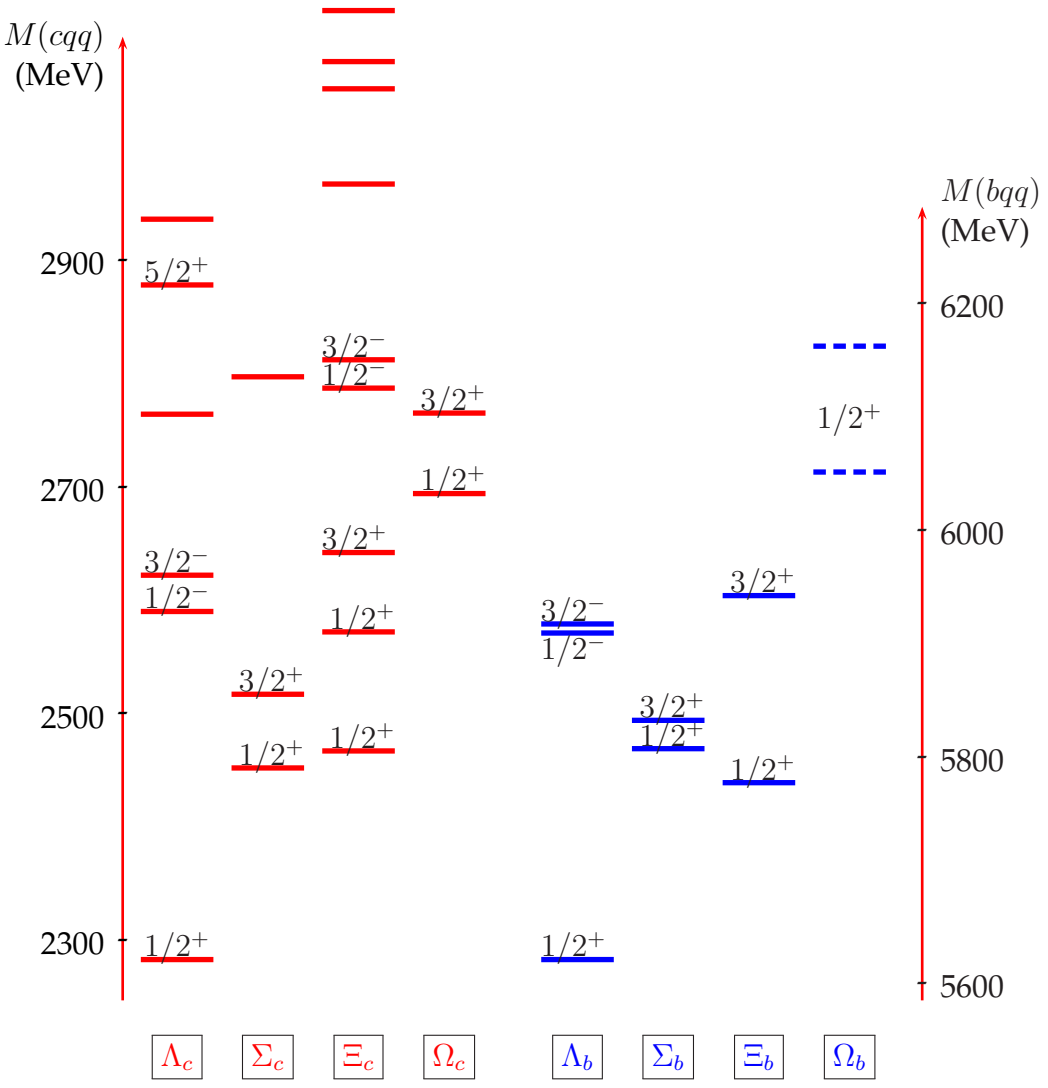

Figure 15: Comparison of (cqq) and (bqq) experimental spectra. Note that some of the quantum numbers have not been established, and are the quark model predictions

communications, see, e.g., [173], it is important to detect double-charm baryons. We also advocate that one should look for double charm baryons, or even, in the same experiments, at "ordinary" double-charm baryons and possible double-charm mesons to be discussed in the next chapter. The production of two $c\bar{c}$ pairs in e^+e^- collisions occurs at a rate that is somewhat larger than expected. It has been used to analyse the mass spectrum of $e^+e^- \to J/\psi + X$, where X is mainly $c\bar{c}$, thanks to the OZI rule. This led to the discovery of $\eta_c(2S)$. One should reasonably expect that sometimes the two pairs hadronise as $(ccq) + (\bar{c}\bar{q}\bar{q}) + (\bar{c}q) + \cdots$.

The dynamics of (QQq) baryons is rather rich, with, in the same hadron, the relativistic motion of a light quark around a coloured source, as in D mesons, and the slow motion of two heavy quarks, as in charmonium. The (QQq) baryons were listed in the pioneer paper by Gaillard et al. on charm physics [52]. The first detailed study in potential models was given in [174] and has been improved by several authors, see, e.g., [175]

At first sight, a quark–diquark approximation seems tempting. Indeed, the average distance between the two heavy quarks, r_{QQ}, is smaller than the distance r_{Qq} between one of the heavy and the light one. However, if the first excitations occurs within the QQ subsystem, and in the quark–diquark picture, one should reconstruct a new diquark for each of them.

An alternative approach is the Born–Oppenheimer approximation, as done for $H_2{}^+$ in atomic physics. For a given $\boldsymbol{x}$, one estimates the light-quark energy and deduce the effective QQ interaction[12]. If this procedure is applied to a simple quark model, one can compare the results to the exact numerical solution of the three body problem [174], and it turns out that the Born–Oppenheimer approximation is extremely accurate[13]. Now, the effective QQ potential of (QQq) can be calculated on the lattice.

The problem is therefore where are the first excitations of this system. The question has been raised in several recent papers, for instance Cohen, Roberts, etc. [176,177]. The answer governs the strategy to be adopted for these systems.

If the (QQ) subsystem enters the regime where the Coulomb interaction $\propto \alpha_s(M)/r$ dominates, then the spacing will behave as $\alpha_s^2\, M$ (not $\alpha_s\, M$ as suggested by a misprint in [177]). Hence the diquark is frozen in its ground state, and the dynamics is that of a light antiquark around a colour $\bar{3}$ source. This is very similar to the physics of flavoured mesons $(\bar{Q}q)$, and this symmetry is promising [176,177].

However, the Coulombic limit of heavy-quark spectroscopy is rather elusive, as noted years ago by [178]. For instance, if one uses the simple quarkonium potential (10) and study the level spacing (2S)–(1S) of $(Q\bar{Q})$ as a function of the quark mass M, one finds that it decreases for $M \lesssim 6$ and increases for $M \gtrsim 6\,\text{GeV}$. So, for the double charm baryons, the first levels are probably excitations in the diquarks, and can be described in the Born–Oppenheimer limit with an effective QQ interaction.

[12] One actually works with the Jacobi variables and thus the recoil effects are properly included

[13] With an energy slightly below the exact one, unlike a variational approximation

4.9 Triple heavy flavour

A classic introduction is a paper by Bjorken [179] who named this sector "the ultimate deal of baryon spectroscopy". This is the analogue of charmonium for baryons, i.e., a sector where the heavy-quark dynamics is probed without any complication due to light quarks. There are already several predictions for the lowest states. Of course, (bbb), (bbc), etc., states are expected as well.

One of the issues is whether (ccc) has s normal hierarchy of excitations, with first the orbital excitation with negative parity, and next the radial excitation which is the analogue of the Roper resonance. For light baryons, an anomalous ordering is observed, probably due to the chiral dynamics of light quarks, which cannot be mimicked by a static, spin-independent potential

4.10 Spin splittings

As for mesons, there are different possible approaches, and some compromises.

Thanks to the charismatic role of Isgur and Karl, the chromomagnetic interaction is the best known mechanism to explain the spin-splittings of ground state baryons. It even works in sector where it should be prohibited. Take $\rho - \pi$: it gets a colour factor $-16/3$, a spin factor 4 ($\boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2 = +1$ for triplet and -3 for singlet), and $|\phi(0)|^2 = \langle\delta^{(3)}(\boldsymbol{r})\rangle$ and results in about 0.6 GeV. For $\Delta - N$, the colour factor is $-8/3$, the spin factor 6, and $\langle\delta^{(3)}(x)\rangle$ is probably reduced, as the wave function is more extended. Hence a value $\Delta - N$ of about 0.3 GeV is roughly compatible.

The systematics of hyperfine splittings in a model based on

$$V_{ss} = \sum_{i<j} \frac{2}{3} \frac{\alpha_s}{m_i\, m_j} \frac{2\,\pi}{3} \delta^{(3)}(\boldsymbol{r}_{ij})\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j \ , \tag{59}$$

was remarkable. See, e.g., [114,139,180,181]. In particular, the chromomagnetic interaction, with its very specific $(m_i\, m_j)^{-1}$ dependence explains why the Λ is ligther than Σ, a long standing puzzle. See the paper by the Orsay group (Le Yaouanc et al.). This approach has been regularly updated [150,166,182–185] See, also, the interesting historical remarks by Lipkin [186], in particular on the pioneer contribution by A. Zakharov.

The chromomagnetic model of hyperfine splittings has been used to make predictions for heavy baryons. It was for instance predicted that the $\Sigma_Q - \Lambda_Q$ splitting will remain large as the mass of the heavy quark increases, while $\Sigma_Q^* - \Sigma_Q$ will decreases. See, e.g., [150], and for a recent upgrade in the beauty sector, [166].

For the fine structure, the situation is less convincing. We have seen that the fine structure of mesons requires a sizeable spin-orbit interaction, which explains the ordering $^3\mathrm{P}_0 < {}^3\mathrm{P}_1 < {}^3\mathrm{P}_2$. This spin-orbit potential is explained as to be resulting from a partial cancellation between the positive spin-orbit of the vector Coulomb potential and the negative spin-orbit of the scalar confinement. It has been argued [139,187] that the cancellation might be more effective for baryons, but still the complete vanishing of the spin-orbit force is not understood.

There is definitely a problem with the spin-orbit splittings in baryons, which are usually very tiny, at least smaller than expected by extrapolation of meson spectroscopy, but sometimes surprisingly large. For instance the $N(1520) - N(1535)$ difference (with $J^P = (1/2)^-$ and $J^P = (3/2)^-$, respectively) is small, but $(\Lambda(1520) - \Lambda(1405)$ is large. So, a strategy initiated by Isgur and Karl consists of setting to zero the spin-orbit forces in baryon, and to call for contributions beyond the simple quark model for the exceptions.

Another approach, also present in the phenomenology of mesons, relies on the role of hadron loops. See, for instance, [188,189]. For sure, the $\Delta \leftrightarrow \pi N$ coupling contributes to the $\Delta - N$ spacing. But the best advertised splitting is between $\Lambda(1405)$ and $\Lambda(1520)$. Both are negative-parity excitations of the ground-state Λ, with isospin $I = 0$. The $\Lambda(1405)$ has $J^P = (1/2)^-$, and $\Lambda(1520)$ has $J^P = (3/2)^-$. Hence, in the quark model, they correspond to the same spin structure as the ground state Λ, a light diquark with spin $s_q = 0$ giving a total quark spin $s = 1/2$. The motion of the s quark with respect to the (ud) diquark has $\ell_y = 1$. Thus there are two possibilities, $J = 1/2$ and $J = 3/2$. The vicinity the $\bar{K}N$ threshold is presumably responsible for this unusually large separation $\Lambda(1405)$ and $\Lambda(1520)$. The pioneer here is Dalitz [190].

A third possibility consists of modifying the part of the quark dynamics dealing with the spin effects. For instance an instanton-induced interaction, inspired but the work of t'Hooft, was adapted by the Bonn group [138]. This was rather successful. Even more advertised is the class of models with a kind of pion-exchange between quarks inside the same hadron. This is named "Goldstone-boson exchange".

Interesting models have been built were both chromomagnetism and Goldstone-boson exchange contribute to the spin splittings.[14]

4.11 Convexity properties

There are not many exact results on the three-body problem. Everything becomes rather involved when going from two to three constituents. For instance, if you consider three distinguishable particles in a local potential, it is not obvious that the lowest energy for each spin J is an increasing function of J.[15]

Among the simple results, one may mention: the energies decrease if any mass m_i is increased; ...

One point that was rather debated is the convexity as a function of the inverse masses. We have seen in the two-body problem that (the symbol $(..)$ is meant for the ground state energy) that $(m, m) + (M, M) \leq 2(m, M)$. Is is thus tempting to generalise as

$$(m, m, m') + (M, M, m') \leq 2\,(m, M, m')\ , \tag{60}$$

for any value of the "spectator" quark of mass m'. This was conjectured [150, 191].

[14] I remember a very interesting discussion with Glozman on this issue. Once a pion is emitted, the quark spin is flipped and thus the intrinsic spin–spin interaction at the quark level is modified. So it is perhaps not so easy to superpose within the same model Goldstone exchange and gluon exchange.

[15] I thank André Martin and Victor Mandelzweig for several discussions on this point.

After some investigation, it turns out that the rule remains true for reasonably-shaped potentials, but that there are counterexamples involving very sharp confinement and large mass ratios [192,193].

The lesson of (60) is that heavy quarks like to cluster together.

4.12 Link between meson and baryon spectroscopy

4.12.1 The baryon potential

In the late 70s, there were clearly experts on baryon spectroscopy, obviously not restricted to heavy quarks, and on the other side, experts on quarkonia. But the question was already addressed of the link between the quark–antiquark interaction in mesons and the three-quark potential in baryons.

One-gluon-exchange, or more generally, any colour-octet exchange gives a factor $1/2$ for the interaction of a pair of quarks in baryons as compared to the quark–antiquark interaction in mesons. This is the analogue of the (-1) factor for one-photon exchange in e^+e^+ as compared to e^+e^-, or the Fermi–Yang rule G for the exchange of mesons, between NN and $N\bar{N}$, where G is the G-parity of the meson or set of mesons.[16]

It is thus reasonable to try, as an educated guess, to apply the same "1/2" rule to the whole potential. After all, when a quark in a baryon, say quark $\#1$, is well separated from the two others, $\#2$ and $\#3$, it feels a colour $\bar{3}$ at separation $\boldsymbol{r} \simeq \boldsymbol{r}_{12} \simeq \boldsymbol{r}_{23}$, exactly as a quark feeling an antiquark at separation $\boldsymbol{r}$, and the "1/2" rule reproduce perfectly this equivalence.

This leads Stanley and Robson, Greenberg, Cohen, Lipkin, etc. [194–196], to discuss the merits of this rule, and to propose a combined phenomenology of both meson and baryon sectors. I remember once Martin making an adventurous extrapolation of his $(c\bar{c})$ and $(b\bar{b})$ potential down to $(s\bar{s})$ [64]. In the good old days of Inquisition, you would be burned at the stake for much less, because $(s\bar{s})$ is anything but non-relativistic. Nevertheless, the agreement was good. I tried the "1/2" rule, and got the mass of $\Omega^-(sss)$ within just a few MeV [197]. There are certainly many regularities in the hadron spectrum that are mimicked by potential models, but the reason of not fully understood.

The main issue is whether the interaction in baryons is pairwise. In nuclear physics, the interaction among nucleons is not pairwise: current potentials fitting the nucleon–nucleon scattering data and the properties of the deuteron underestimates the binding of few-nucleon systems. Three- and even four-nucleon forces have to be introduced. In atomic physics, there are empirical potentials between for instance two He atoms (Asiz, etc.). They can be reproduced by a (complicated but feasible) Born–Oppenheimer calculation, where the energy of four electrons around two fixed α particles is properly minimised. Then the interaction energy of three He atoms can be estimated, by minimising the energy of six electrons around three fixed α's. The interaction contains three-body terms, which are not very important in practice, but are there.

[16] The Fermi–Yang rule relates the potentials with the same isospin, for instance, pp and $p\bar{n}$.

Similarly, any physical interpretation of the linear confinement in mesons leads to a generalisation for baryon which is of the three-body type. See, for instance, [149, 198–202], but many important papers have been probably omitted. In early days, some approaches were a little empirical. For instance, in [149], the same variant of the bag model used for quarkonia and hybrid mesons was applied to a bag containing three heavy quarks surrounded by their gluon field. At very large separations, the bag that minimises the gluon energy is Y-shaped, with three tubes joining at the centre of the baryon. Today, the potential of baryons is derived in lattice QCD and even in AdS/QCD, as will be explained by the other lecturers.

The main result is that if you consider that $V(r) \sim b\, r$ in (10) corresponds to a gluon tube of constant section whose linear behaviour $\propto r$ minimizes the energy, the analogue for a baryon is a Y-shape set of three tubes whose cumulated length minimises the energy, i.e.,

$$V(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) = b \min_J (r_{1J} + r_{2J} + r_{3J}) \,, \tag{61}$$

where the junction J is adjusted to minimise the sum. The solution is well known. For a flat triangle, with, say, $\angle 213 > 120^\circ$, then J coincides with the quark #1, other wise, J sees each side at 120°, as shown in Fig. 16.

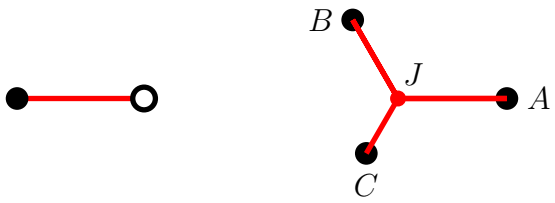

Figure 16: The string linking the quark to an antiquark in a meson (left) or the three quarks of a baryons has a minimal length.

As a French lecturing in Italy, I cannot resist mentioning here the work of Fermat and Torricelli, and of Napoleon. The two first names will not be too much a surprise, and, indeed, Torricelli had an interesting correspondence with several scientists, including Fermat. Napoleon Bonaparte invaded Italy, among other countries, and his troops behaved wildly, in particular in Naples, and the following sentence is reported "Non tutti i francesi sono ladri, ma buona parte sì". Napoleon was also a fine mathematician, and his theorem on triangles is relevant for the confinement of baryons.

The problem of the minimal cumulated distance to three given points was solved by Fermat and Torricelli. A possible construction is shown in Fig. 17.

One may also remark that the circumcircles of the external triangle such as BCA' intersect in J. The construction is also that of Napoleon's theorem, which states that the centres of these external triangles form an equilateral triangle. See Fig. 18. This is an interesting example of symmetry restoration starting from an asymmetric triangle.

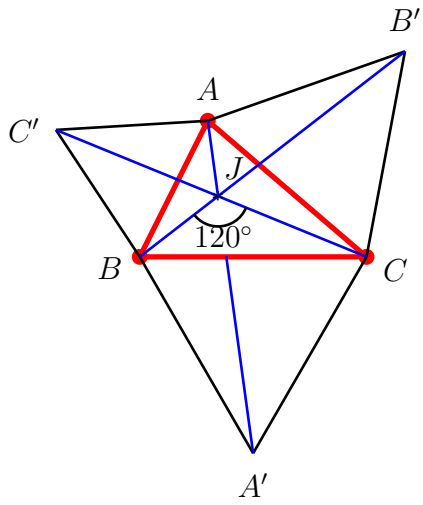

Figure 17: If the quarks are located at A, B and C, and if A' is the third vertex of an external equilateral triangle built on the side BC, and similarly for B' and C', then the straight line AA', BB' and CC' intersect at the Fermat–Torricelli junction. Moreover, the minimal sum of distances, $Y = JA + JB + JC$ is equal to $Y = AA' = BB' = CC'$

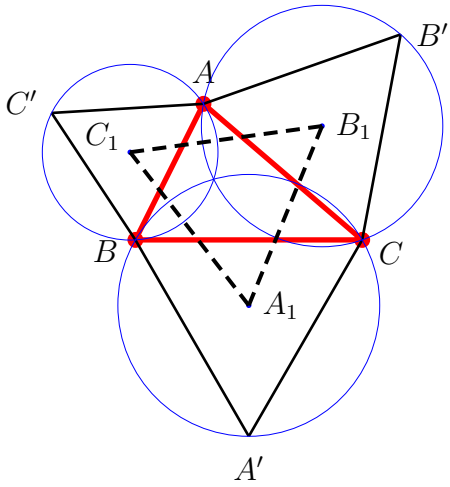

Figure 18: The Fermat–Torricelli junction is at the intersection of the circumcircles of the external equilateral triangles which are at the basis of Napoleon's theorem.

4.12.2 Inequalities between meson and baryon masses

Presumably, the short range contributions are pairwise and fulfil the 1/2 rule, and the long range linear confinement is given by the Fermat–Torricelli minimal length Y (times the string tension). It is easy to check that the latter is bound by (p is the perimeter)

$$\frac{p}{2} \leq Y \leq \frac{p}{\sqrt{3}} \,, \tag{62}$$

(the lower bound is saturated for a flat triangle, the upper one for an equilateral triangle), so that for the whole baryon potential,

$$V \geq \frac{1}{2}[v(r_{12}) + v(r_{23}) + v(r_{31})] \,. \tag{63}$$

In the 60s, physicists working on few-nucleon systems, or on quantum systems under gravitational interaction, have derived inequalities between three-body and two-body energies. In our case,

$$V \geq \frac{1}{2}[v(r_{12}) + v(r_{23}) + v(r_{31})] \,. \tag{64}$$

and hence for the Hamiltonians

$$H_3 = \frac{\boldsymbol{p}_1^2}{2\,m} + \cdots + V \geq \frac{1}{2}\left\{\left[\frac{\boldsymbol{p}_1^2}{2\,m} + \frac{\boldsymbol{p}_2^2}{2\,m} + v(r_{12})\right] + \cdots\right\} \,. \tag{65}$$

Since the minimum of the sum is larger than the sum of minima, the above inequality immediately implies $E_3 \geq (3/2)\,E_2$, or after adding the constituent masses [203],

$$2\,M(qqq) \geq 3\,M(q\bar{q}) \,. \tag{66}$$

In other words, the matter is heavier per quark in the baryon form than in the meson form. The inequality (66) has been generalised and improved in several ways. See, e.g., [204–206] for some details and references to the pioneer works, in particular by Hall, Post, Fisher, Ruelle, Lévy-Leblond, etc.

In the form $(qqq) + (\bar{q}\bar{q}\bar{q}) \geq 3\,(q\bar{q})$, the inequality (66) implies that, at least within a simple quark model, baryon–antibaryon "annihilation" (more precisely rearrangement) into three mesons is energetically allowed, even at rest.

On the other hand, if the mass ratio becomes large enough, the inequality is inverted, and becomes

$$(\bar{Q}\bar{Q}\bar{Q}) + (qqq) \leq 3\,(\bar{Q}q) \,, \tag{67}$$

meaning that a triply-flavoured antibaryon would not annihilate into ordinary matter. The critical value of M/m is model dependent. There is in (67) another manifestation of the tendency of heavy constituents to cluster together, as seen already in (37).

"Why do it the easy way when you can do it the hard way?" was the favourite watchword of a popular TV series in the late 60s. An alternative (and not so rigorous)

way of understanding $(qqq) + (\bar{q}\bar{q}\bar{q}) \geq 3\,(q\bar{q})$ is as follows. The reasoning will be useful in the section about multiquarks.

Consider the case where the interaction is pairwise and the 1/2 rule is exact. Otherwise replace some equalities by inequalities. Both $(qqq) + (\bar{q}\bar{q}\bar{q})$ and $(q\bar{q}) + (q\bar{q}) + (q\bar{q})$ systems can be artificially considered as governed by a 6-body Hamiltonian (for more mathematical rigour, one can add a very weak overall harmonic confinement that makes all levels discrete and normalisable)

$$H_6[\{g_i j\}] = \sum_i \frac{\boldsymbol{p}_i^2}{2\,m} + \sum_{i<j} g_{ij}\, v(r_{ij})\ , \qquad \sum_{i<j} g_{ij} = 3\ . \tag{68}$$

Let $E_6[\{g_i j\}]$ denote the ground state. From the variational principle, it is maximal for the fully symmetric case where $g_{ij} = 1/5\ \forall i, j$. The baryon-antibaryon case corresponds to a set of coefficient ("*" means "times") $\{g_{ij}\} = \{6 * (1/2), 9 * 0\}$, and the three-meson configuration to $\{g_i j\} = \{3 * 1, 12 * 0\}$. The latter has the same mean value 1/5 as the former, but a much larger variance, about 0.17 vs. about 0.06, and this makes its energy lower.

5 Multiquarks

Le plus grand dérèglement de l'esprit,
c'est de croire les choses parce qu'on veut qu'elles soient,
et non parce qu'on a vu qu'elles sont en effet. [17]
Bossuet

5.1 Introduction

We are now entering a delicate chapter. An atomic physicist could not feel satisfied with only the hydrogen atom and its variants, and the H^- and $H_2{}^+$ ions, or a nuclear physicist with the deuteron, tritium and ^{3}He: they have to understand why some small systems are less easily bound than others, and to explore more complicated structures. Similarly, one has to know whether there exist hadrons beyond ordinary mesons and baryons. Besides multiquarks, several systems have been considered, either exotics, i.e., with quantum numbers that cannot be matched by $(q\bar{q})$ nor (qqq), or crypto-exotics, with the same quantum numbers but different internal structure. The latter are more difficult to identify: one should first detect the exhaustive spectrum of ordinary hadrons, and look at super-numerous peaks, but the signature can be hidden by mixing of ordinary and extra-ordinary hadrons.

[17] The biggest disorder of the spirit, it is to believe things because we want that they are, and not because we saw that they are indeed.

5.1.1 Glueballs

They used to be fashionable, and motivated several experiments. In a glueball, a few constituent gluons experience a confining interaction. There is a considerable phenomenology of glueballs, and many predictions based on lattice QCD, with noticeable fluctuations along the years [207].

5.1.2 Hybrid mesons

At first sight, this is just another piece of the folklore, combining mesons and glueballs. However, it acquired some ground.

Consider $H_2{}^+$ in the Born–Oppenheimer limit, which is a very good approximation. At given pp separation r, one estimates the minimum of the electronic energy, which, when supplemented by the direct Coulomb repulsion, built the effective pp potential. In the Schrödinger equation, this potential gives the ground-state and the first orbital and vibrational excitations. If one takes the second electronic energy, one gets another effective potential generating a second set of levels.

Now, replace the protons by a quark and an antiquark, and the electron by the gluon field, you first get the ordinary quarkonium and next the hybrids. Pioneer papers are [208, 209]. The quark–antiquark potential within an excited gluon field was explicitly estimated in a variant of the bag model [210], where gluons are treated classically and confined in volume whose shape and size is optimised: a $Q\bar{Q}$ potential is derived, very similar to the Coulomb-plus-linear one, and a second potential is obtained, with a repulsive Coulomb interaction and a confinement smoother than for quarkonia. See Fig. 19.

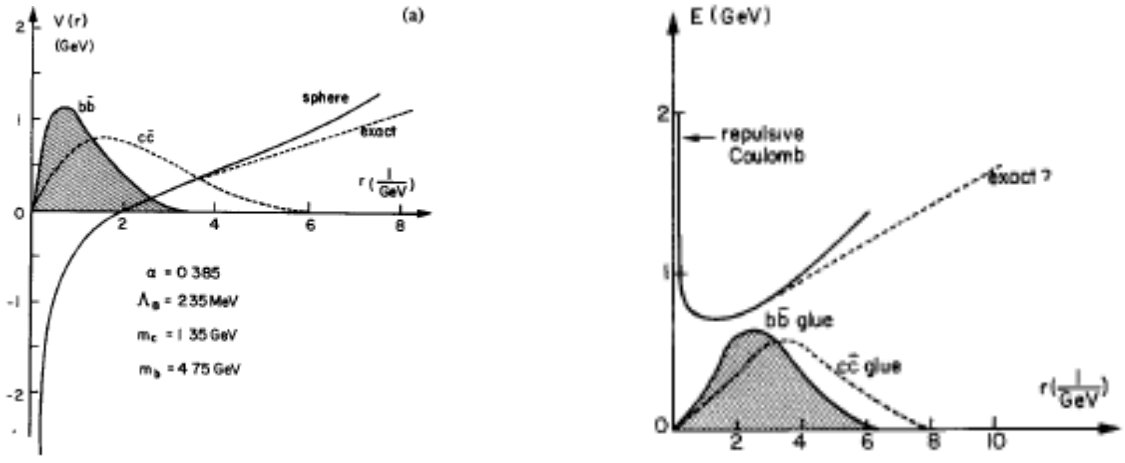

Figure 19: Left: ordinary $Q\bar{Q}$ potential, right: effective $Q\bar{Q}$ potential for hybrids.

5.1.3 Hadron molecules

It is regularly rediscovered that the Yukawa mechanism leading to the nuclear forces is by no means restricted to the nucleon–nucleon interaction. Any particle containing light quarks or antiquarks can emit or absorb light mesons and thus enter a nuclear-type of interaction.

The hidden-charm sector is no exception. Already in 1975, Iwazaki suggested that ψ' could be $(c\bar{c}q\bar{q})$ [211], where q denotes u or d. In 1976, Voloshin and Okun [212] suggested the existence of molecules made of a charmed meson and an anticharmed meson. De Rujùla, Georgi and Glashow [79] were intrigued by one of the recurrence of J/ψ, with the same quantum numbers $J^{PC} = 1^{--}$, whose branching ratios into $D\bar{D}$, $D^*\bar{D} + \text{c.c.}$ and $D^*\bar{D}^*$ were anamalous as compared to naive spin counting. They suggested a $D^*\bar{D}^*$ structure for the $\psi(4.03)$. In fact, the anomalies in the branching ratios were explained by the node structure of the state: roughly speaking, a two-body decay calls for a range of relative momentum, and the decay is either suppressed or enhanced if this momentum is near a zero or a maximum of the probability in momentum space.

More recently, Törnqvist [213, 214], and also, Ericson and Karl [215], Manohar and Wise [216], etc., stressed that the one-pion exchange between two charmed mesons or between a charmed and an anticharmed meson, can be attractive. This potential is somewhat weaker than the potential between a proton and a neutron, but of the same order of magnitude. It is experienced by heavy particles, and thus could lead to binding. Remember that for an Hamiltonian $H = \boldsymbol{p}^2/m + g\,V(r)$, where $V(r)$ is attractive or contains an attractive part, the existence of a discrete spectrum depends on $m\,g$.

The discovery of the $X(3872)$ just at the $D^*\bar{D}$ threshold was considered as the success of the prediction by Törnqvist and others. Some further measurements have confirmed the existence of this state. However, some decay properties, such as $X \to \psi(2S) + \gamma$ larger than $X \to \psi(1S) + \gamma$, suggest the structure of a radially excited P-state. Presumably both charmonium and molecular components exist in the wave function. The question now is whether this is just a charmonium state modified by the coupling to its nearest threshold, or whether there are really two separate states in the spectrum, one mainly $2\,^3\mathrm{P}_2$ state of charmonium, another mainly a $D^*\bar{D} + \text{c.c.}$ molecule, both with $J^{PC} = 1^{++}$.

I cannot review the literature on molecules, especially the papers based on effective theories. A superficial reading gives the feeling that for any pair of hadrons, one can write up an effective Lagrangian that will lead to a molecule. The approach based on conventional nuclear forces is more predictive. Remember the Yukawa interaction in a S-wave contains a spin–isospin factor $\boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2\,\boldsymbol{\tau}_1.\boldsymbol{\tau}_2$, and thus is either attractive or repulsive and varies by a factor 1:9 in magnitude. After the discovery of the $X(3872)$, the literature exploded. See, e.g., the reviews by Swanson [217] and by Nielsen et al. [218]. Recent contributions include [219, 220]

5.2 Baryonium

In the 70s, bumps were tentatively seen in antiproton cross sections, and peaks in inclusive γ-ray spectra of $\bar{p}p \to \gamma X$. The generic name "baryonium" was invented for new mesons preferentially coupled to the baryon–antibaryon channels. For a review of the early results, see [221].

A molecular picture of baryonium was proposed, e.g., by Shapiro and his collaborators [222], Dover et al. [223–225], etc. The interaction is given by meson-exchange (Fermi and Yang [226] have given a simple rule to transform meson exchange into a particle–particle system to the corresponding particle–antiparticle system) at large distances, and exhibit a noticeable attraction in some partial wave. This suggests the possibility of $N\bar{N}$ bound states, in the same way that the exchange of mesons leads to a bound state of two nucleons. The role of annihilation remains however uncertain in this approach.

A four-quark picture of baryonium was also elaborated, and in fact was even anticipated by arguments based on the so-called "duality", whose explanation is beyond the scope of this review. See, e.g., Veneziano and Rossi [221] and Jaffe [227, 228]. Unfortunately, if the authors of the above references were relatively careful in their predictions, this was not always the case with the followers, who produced dozens of baryonia and generalisations.

Chan H.M., Høgaasen and their collaborators speculated about the role of the internal colour degrees of freedom [229].[18] For $(q^2\bar{q}^2)$, they named "true" baryonium a state where the quarks are in colour $\bar{3}$ and the antiquarks in colour 3. Such a state is thus easily coupled to the baryon–antibaryon channels but can be prevented of annihilating too fast if the quarks are separated from the antiquarks by some angular-momentum barrier.[19] States with colour structure $6\bar{6}$ were also envisaged and named "mock" baryonia. It was suggested they could be rather narrow. Of course, what was not clear in this approach is why four-quark states should cluster in the form of a diquark and an antidiquark with such colour structure. See, e.g., [230].

No baryonium survived better experimental scrutiny, in particular with the intense and cooled antiproton beam of the LEAR facility at CERN. More recently, hadrons containing heavy quarks have been observed decaying into baryon + antibaryon + $\cdots$, and enhancements are seen in the spectrum of the invariant mass $p\bar{p}$, probably due to a strong final-state interaction. So, the saga of baryonium is perhaps not fully over.

Note also that the mechanisms invented to explain the tentative baryonium states were applied to other configurations. For instance, Strottmann [231], Sorba and Høgaasen [232], and De Swart et al. [233] studied baryon states with four quarks and one antiquark (the name "pentaquark" was used later). Other studies dealt with $(q\bar{q} - q\bar{q})$ [234] or with dibaryons [235].

[18] I thank Chan H.M., H. Høgaasen, P. Sorba and B. Nicolescu for numerous discussions

[19] Note that the structure $(qq - \bar{q}\bar{q})$ for angular momentum $\ell > 0$ is not demonstrated, it is assumed

5.3 Chromomagnetic binding

5.3.1 The chromomagnetic interaction

As mentioned earlier, an appealing aspect of the quark model for mesons and baryons is the systematics of hyperfine effects such as $J/\psi - \eta_c$ or $\Delta - N$. Lipkin [186, 236] likes to stress that Sakharov, though isolated, had interesting remarks on the subject. A popular reference is the paper of DGG [114], where hyperfine forces among quarks are described in analogy with the Breit–Fermi interaction in atomic physics and attributed to one-gluon-exchange. See also [115].

This mechanism was used in the bag model [237], and in several variants of potential models, with a component

$$V_{ss} = \sum_{i<j} \frac{K}{m_i\, m_j}\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\, \tilde{\lambda}_i.\tilde{\lambda}_j\, v_{ss}(r_{ij})\ , \tag{69}$$

where v_{ss} is either a contact term, to be treated at first order, or a regularised contact term, to be inserted in the wave equation. We already stressed the importance of the strength being proportional to $(m_i\, m_j)^{-1}$, for instance for the $\Sigma - \Lambda$ difference, but now we will first consider the case of equal masses. For angular momentum $\ell > 0$, there are additional terms, in particular a tensor component.

5.3.2 The dibaryon *H*

At the end of 1976, Jaffe [238] studied the $(ssuudd)$ system in the SU(3)$_{\rm F}$ limit, assuming that $\bar{v}_{ss} = \langle v_{ss}(r_{ij})\rangle$ has the same value for all the pairs, thus concentrating on the properties of the operator $\mathcal{O} = \sum \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\, \tilde{\lambda}_i.\tilde{\lambda}_j$ in this dibaryon system, as compared to the baryons. He discovered a very intriguing coherence, namely that for some configurations $\langle\mathcal{O}\rangle$ can be attractive and *larger* (in absolute value) than its cumulated value for two baryons constituting the threshold! This is really remarkable. For the positronium molecule and for two positronium atoms constituting its threshold, both described by a potential $\sum\, g_{ij}\, r_{ij}^{-1}$, we have the *same* cumulated strength $\sum g_{ij} = -2$.

Using for $\bar{v}_{ss}$ the same value as in the light baryons, Jaffe predicted a "di-lambda", named H, bound by about 150 MeV below the baryon–baryon threshold. In the SU(3)$_F$ limit considered by Jaffe, the N, Λ, Σ or Ξ receives from (69) a downward shift of 150 MeV (this is half of the $\Delta - N$ splitting), and thus the $\Lambda\Lambda = N\Xi$ thresholds benefit from about 300 MeV. Thus the H receives an additional 150 MeV as compared to this threshold. This was a dramatic change in the physics of multiquarks. Earlier speculations, such as baryonium, proposed metasable states, lying above their lowest threshold, but perhaps not too broad. The H, as initially proposed, was stable against any dissociation, and thus decaying weakly.

As the MIT physicists are very convincing and powerful, the H was searched for in about 20 experiments.[20]

[20]For comparison, the pentaquark analogue discussed in the next section was searched in a single

Unfortunately, it was later realized that the first calculation by Jaffe needed some improvements, namely [239–242]: *i)* treat the spin-independent interaction (instead of assuming that (q^6) and (q^3+q^3) starts from the same line before switching on the hyperfine effects, *ii)* consider SU(3)$_{\rm F}$ breaking that spoils the coherence of the configurations $\Lambda\Lambda$, $N\Xi$, etc., , and *ii)* use for $\bar{v}_{ss}$ values consistently computed from 6-body wave functions. Each effect dramatically reduces the binding, and the H becomes eventually unbound, though there is some model dependence. Recent lattice simulations [243,244] leaves the door open.

5.3.3 The heavy pentaquark P

In 1987, Gignoux et al. [245] and Lipkin [246] realised independently that the scenario for the H is repeated in a baryon configuration involving a heavy antiquark and four light quarks.[21]

Let us consider a configuration $(\bar{Q}qqqq)$ in the limit m_Q for the antiquark and exact SU(3)$_F$ for the quarks. If the quarks have spin 0 and form a triplet of flavour. This means $(uuds)$, $(udds)$ or $(udss)$. Then the chromomagnetic operator $\mathcal{O}=\sum \boldsymbol{\sigma}_i.comp\boldsymbol{\sigma}_j\,\tilde{\lambda}_i.\tilde{\lambda}_j$ has an expectation value which is negative and twice larger than for Λ. This means that if you compare the pentaquark[22] $P(\bar{Q}uudqs)$ to its thresholds $(\bar{Q}q)+(qqq)$, and adopt the same short-range correlation factor $\bar{v}_{ss}$, you find the pentaquark bound by aboundkarl 150 MeV. This pentaquark was searched for in an experement at Fermilab [247,248], but the results were not conclusive.

This pentaquark was further analysed, and its was found, as for H, that the corrections tend to spoil the binding. They are: breaking of SU(3)$_F$, finite mass for the heavy antiquark, self-consistent estimate of $\langle v_{ss}(r_{ij})\rangle$, or even better, inclusion of v_{ss} in an accurate five-body calculation, etc. See, for instance, [242,249,250]

5.3.4 Other configurations

The chromagnetic mechanism remains fascinating, even so it does not bind as easily as first thought. It contributes to the structure of the X, Y, Z states [251], and to the short-range hadron–hadron interaction. Leandri and Silvestre-Brac have scanned the configurations where interesting coherences occur [252, 253]. Then they investigated whether the stability suggested by the chromomagnetic operator survived a more detailed calculation, with mostly negative results.

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experiment, and is vaguely in the agenda of the COMPASS experiment at CERN. *"Selon que vous serez puissant ou misérable, les jugements de cour vous rendront blanc ou noir."* (According as you're feeble, or have might, High courts condemn you to be black or white) Les animaux malades de la peste, Jean de la Fontaine

[21] Lipkin, in fact, had some precursor idea in this direction at Baryon 80 in Toronto. Anyhow, I remember Lipkin arriving at Les Houches for the Workshop on the "Elementary Structure of Matter", handing me a freshly issued preprint, myself welcoming him with a copy of our latest manuscript, and our astonishment when we realized that both papers were dealing with the same exotic baryon!

[22] The word was adopted at this occasion

5.4 Binding by central forces

In a novel by Alphonse Daudet, one finds this sentence: " ...il ferma la porte à double tour. Malheureusement, il avait oublié la fenêtre ..." (He double locked the door, but forgot the windows). Similarly, if the chromomagnetism cannot bind multiquarks, why not try chromoelectricity?

Believe it or not, it works, and there is an amusing side effect, or I should perhaps say, a miracle: while the literature on multiquarks is usually a jungle, of controversed, here a consensus has been reached that some configurations should support stable tetraquarks, i.e., states below the threshold for their dissociation into two mesons.

I will first summarise the results obtained from an empirical pair-wise ansatz for the multiquark interaction, and in the next subsection, explain the changes brought by an improved modelling of the dynamics.

5.4.1 The additive model

As explained in Sec. 4.12, the potential for mesons can be extrapolated to baryons by the "1/2" rule, i.e., $V = [v(r_{12}) + v(r_{23}) + v(r_{31})]/2$, where v is the quarkonium potential. It can be read as the exchange of a colour octet in the t-channel. Similarly, in nuclear physics, there are isoscalar exchanges, leading to an isospin-independent interaction, and isovector exchanges, which get a $\boldsymbol{\tau}_1.\boldsymbol{\tau}_2$ factor, such a pion-exchange. The analogy is of course delicate, as local and global symmetries are not on the same footing. Nevertheless, the multiquark phenomenology in simple constituent models has been very often based on the assumption that

$$V = -\frac{16}{3} \sum_{i<j} \tilde{\lambda}_i.\tilde{\lambda}_j \, v(r_{ij}) \, , \tag{70}$$

which reduces to the 1/2 rule in the case of baryons. Here is the version for quarks. For an antiquark, $\tilde{\lambda}_i \to -\tilde{\lambda}_i^*$

This additive model leads to coupled equations, using either a $(q\bar{q}) - (q\bar{q})$ basis with colour 1 or 8 for the pairs, or a $(qq) - (\bar{q}\bar{q})$ basis with internal colour $\bar{3} - 3$ or $6 - \bar{6}$.

Solving the four-body problem is a considerable task, and requires a lot of care. It can be done by variational methods based, for instance on a superposition of Gaussian wave functions, or by hyperspherical expansion.

The hyperspherical expansion consists of considering the n Jacobi variables [23] of a $(n+1)$-body problem, as a unique $3\,n$-dimensional vector, and to use spherical coordinates (ϱ, Ω) to describe it. In the particular case of the harmonic oscillator with equal masses, the potential is isotropic, $V \propto \sum r_{ij}^2 \propto \varrho^2$, and a single radial equation allows one to recover the standard results for this interaction. In general, one deals with an anisotropic potential in the $3\,n$-dimensional space, and thus an infinite set of coupled equations (the same situation would occur for a single paticle in a potential that is not

[23] if necessary rescaled to be associated with the same reduced mass

rotation-invariant). One obtains a rather satisfactory convergence as a function of the number of equations actually included.

5.4.2 Results for equal masses

If this colour-additive potential, without spin-dependent terms, is applied to $(q, q\,\bar{q}, \bar{q})$, no bound state is found below the $(q\bar{q}) + (q\bar{q})$ threshold. Even though such a model is rather primitive, it already indicates that there is no proliferation of narrow multiquarks.

One may wonder why the positronium molecule is stable whereas the $(q, q, \bar{q}, \bar{q})$ is not in such a simple quark model. The generic Hamiltonian for these systems is

$$H_4 = \sum_i \frac{\boldsymbol{p}_i^2}{2\,m} - \frac{[\sum \boldsymbol{p}_i]^2}{8\,m} + \sum_{i<j} g_{ij}\, v(r_{ij})\ , \quad \sum_{i<j} g_{ij} = 2\ , \tag{71}$$

where $v(r) = -1/r$ for the atomic problem, and $v(r) = \lambda\, r + \cdots$ in the quark problem. For the thresholds (atom–atom or meson–meson), the molecule, and the "true" $T = (\bar{3}3)$ and "mock" $M = (6\bar{6})$ colour wave-functions of the tetraquark system, the coefficients are given in Table 4.

Table 4: Coefficients g_{ij} of the potential energy for the threshold, the positronium molecule and the T and M type of tetraquark, average and variance

State \ Pair	12	34	13	24	14	23	$\bar{g}$	Δg
Threshold	0	0	1	1	0	0	1/3	0.22
Ps_2	-1	-1	1	1	-1	-1	1/3	0.89
T	1/2	1/2	1/4	1/4	1/4	1/4	1/3	0.01
M	$-1/4$	$-1/4$	5/8	5/8	5/8	5/8	1/3	0.17

We repeat here in some more detail a reasoning sketched at the end of Sec. 4.12.2 dealing with the comparison of mesons and baryons. Symmetry breaking is known to lower the energy of the ground state. For instance, the linear oscillator $h(0) = p^2 + x^2$ has a ground state at $\epsilon(0) = 1$ which becomes $\epsilon(\lambda) = 1 - \lambda^2/4$ for $h(\lambda) = h(0) + \lambda\, x$. More generally, for $H(\lambda) = H_0 + \lambda\, H_1$, where H_0 is even and H_1 odd, one gets the same lowering $\epsilon(\lambda) \leq \epsilon(0)$ of the ground state, and this is true for symmetries other than parity. This is easily seen by using the variational principle, with the symmetric ground state of H_0 as a trial wave function for $H(\lambda)$.

Thus, if one considers the family of Hamiltonians (71), the ground state energy is maximal if all couplings are equal, i.e., $g_{ij} = \bar{g}\ \ \forall i, j$ and $\Delta g = 0$.

If $H_4 = H_0 + \lambda\, H_1$ with H_0 symmetric and H_1 a given asymmetric potential energy, then the ground state $\epsilon(\lambda)$ is maximum at $\lambda = 0$ and concave. Thus the larger λ for $\lambda > 0$, the lower the energy. And the larger $|\lambda|$ for $\lambda < 0$, the lower the energy.

This is rigorous, so far. But the energy, as a function of λ is nearly symmetric, and for $H_4 = H_0 + \lambda H_1$, the energy $\epsilon(\lambda)$ decreases approximately as $\epsilon_0 - \lambda^2 \tilde{\epsilon}_0$. Furthermore, in the case of (71), H_0 corresponds to the symmetric case $g_{ij} = \bar{g}$, and one can probe different H_1, i.e., different ways of breaking the symmetry among the pairs in the potential: the energy decreases approximately according to the variance Δg of the distribution of the coefficients $\{g_{ij}\}$.

From the values listed in Table 4, one sees that Δg is larger for Ps_2 than for two non interacting atoms. This explains (again, not very rigorously) why Ps_2 is stable in atomic physics. On the other hand, for the T or M tetraquarks, Δg is smaller than for two non-interacting mesons. This suggests that in the simple additive model without spin-dependent terms, there is no stable tetraquark, and the mixing of colour configurations does not improve much the situation. In other word, the tetraquark is penalised as compared to the positronium molecule by the *non-Abelian* character of the theory.

5.4.3 Binding for unequal masses

The lesson of the above subsection is that the tetraquark potential is too symmetric with respect to permutations, as compared to the potential governing the two-meson threshold. So to get binding, another effect is required. We have mentioned chromomagnetism, we shall see below the possibility of improving the modelling of spin-independent forces, to better conform with QCD. There is another effect, which is conceptually simpler: introduce symmetry breaking in the kinetic energy, i.e., mix different flavours in the multiquarks.

As stated earlier, any symmetry breaking lowers the ground-state energy, but does not necessarily improve binding. Usually the threshold benefits more from symmetry breaking, thus the stability deteriorates.

This is the case, for instance, in atomic physics, when one breaks permutation symmetry for $(\mu^+, \mu^+, \mu^-, \mu^-)$. For simplicity, let us do it in the same way in the two charge sectors, i.e., keeping an eigenstate of charge conjugation. This corresponds to (M^+, m^+, M, m^-), and one can use the scaling properties of the Coulomb problem to impose $M^{-1} + m^{-1} = 2\,\mu^{-1}$ without any loss of generality. This corresponds to maintaining the threshold energy constant. What is observed is that the positronium-line molecule is stable for $M = m$, but becomes unstable for $M \gtrsim 2.2\,m$ or $m \gtrsim 2.2\,M$. For a large value of the mass ratio, the ground-state consists of a protonium atom which cannot polarise a positronium atom to attach it. (Of course, some metastability of an hydrogen atom and an antihydrogen atom cannot be exclude, but this is a much more delicate issue.)

The same mechanism renders $(Q, \bar{Q}, q, \bar{q})$ less bound than the equal-mass tetraquark in any simple, flavour-independent, central potential. This is of course rather unfortunate, since the best candidates today, to be discussed in Patricia's lectures following mine, are just in the hidden charm or hidden beauty sector! The mechanism here is presumably long-range forces, such as pion exchange, and spin-dependent forces.

Another possibility if symmetry breaking with two masses consists to give up charge

conjugation and keep particle identity in the two sectors. In atomic physics, the configuration reads (M^+, M^+, m^-, m^-). It is well known that the binding energy of the hydrogen molecule (in units of the threshold energy to have a scale-independent statement) is much larger than that of the positronium molecule. Historically, the hydrogen molecule was studied in the Born–Oppenheimer–Heitler–London limit, starting from $M \to \infty$, while the question of the stability of the positronium was addressed later by Whyleer, Ore, Hylleraas, etc.

The Hamiltonian can be written as follows

$$H(M^+, M^+, m^-, m^-) = H(\mu^+, \mu^+, \mu^-, \mu^-) + \frac{1}{4}\left(\frac{1}{m} - \frac{1}{M}\right)\left[\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 - \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2\right] \ , \quad (72)$$

In (72), with the choice of the inverse of μ averaging the inverse masses, the two Hamiltonians have the same threshold, but the hydrogen molecule benefits from the term breaking charge-conjugation, and thus has a lower ground state! In other words, the stability of the hydrogen molecule is a consequence of the stability of the positronium molecule, *via* the variational principle.

However, the Coulomb character of the potential hardly matters, what is crucial is property that the same potential governs both systems. Thus it is not a surprise that in a simple quark model with flavour independence, one finds that $(Q, Q, \bar{q}, \bar{q})$ becomes stable if the mass ratio M/m is large enough. This was found by Ader et al. [203] and in many subsequent papers, see, e.g., [220, 254–261], and references there.

The question is now how large M/m should be. It depends on the details of the model, it depends how good is the four-body calculation, and whether or not spin-dependent corrections are also added. For long, it was believed that with current pairwise models, $(b, b, \bar{q}, \bar{q})$ was required, where $q = u,\ d$. Two b or not two b? The question was answered by Jang and Rosina [259], who got binding with $Q = c$, using an additive potential model fitting mesons and baryons. This was confirmed by Barnea et al. [261].

5.5 Multiquarks in a Steiner-tree potential

5.5.1 A new confinement for tetraquarks

As already discussed, the pairwise interaction with colour factors is not dictated by QCD. It is a convenient tool for preliminary investigations. For sure, the ansatz looks reasonable for a N-body colour-singlet, when a quark is well separated from the $(N-1)$ remaining constituents: the quark feels the same interaction as in a quarkonium. But there is no fundamental reason why the interaction should be pairwise.

Years ago, Artru [198], followed by many others [149, 199–202, 262], pictured the mesons as a simple string linking the quark to the antiquark, and baryons as three strings, each linking a quark to an intersection named *junction*. In the strong coupling regime of QCD, this corresponds to a flux of gluons linking the quarks. The flux having a constant cross section, the potential grows linearly.

The analogue for tetraquarks of the Y-shape interaction of baryons consists of two terms,

1. A flip-flop interaction, V_f, which is the minimum of $v(r_{13}) + v(r_{24})$ and $v(r_{14}) + v(r_{23})$,

2. A connected Steiner-tree, V_s, which links the quarks to the antiquarks with a minimal length.

The combination of flip-flop and Steiner tree, with a linear behaviour, namely,

$$V_4 = \min(V_f, V_s) \, , \tag{73}$$

thus involving a discrete two discrete minimisations and a minimisation over the continuous variables describing the location of the junction of the Steiner tree. This interaction was proposed by Carlson et al. [263], Vijande et al. [264], and Ay et al. [265]. The model (73) is schematically pictured in Fig. 20

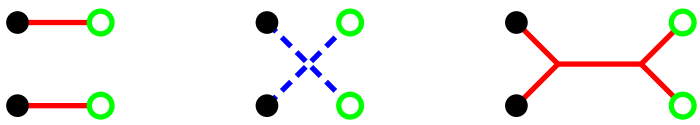

Figure 20: Simple string model for the tetraquark: the potential is the minimum of two meson–meson links (flip–flop) and a connected contribution.

5.5.2 The flip-flop interaction

The flip-flop interaction was proposed in a celebrated paper by Lenz et al. [266], with, however, a quadratic behaviour for $v(r)$. In this paper is discussed the possibility of extending the quark model of ordinary mesons and baryons to the study of multiquark states and of the hadron–hadron interaction.

5.5.3 The connected string

It has interesting properties. In the planar case, it can be constructed by duplicating Napoleon's construction for baryons, see Fig. 21.

In space, the length of the minimal Steiner tree is the maximal distances between two circles. The first one includes all the points making an equilateral triangle with the quarks, the second one for the antiquarks. See Fig. 22. Now if you wish for the best algorithm to estimate the maximal distance between two circles in space, you have to read the reviews on computer-assisted cartoons. The software instructions in this branch of art are such that once a few basic elements are drawn, you are guided to deduce

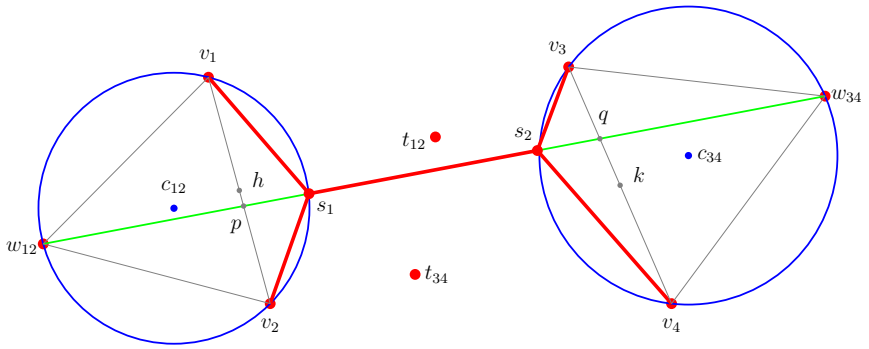

Figure 21: In a plane, the Steiner-tree potential can be constructed as the Fermat–Torricelli minimal sum of distances for baryons. The length of interest is equal to distance between the two auxiliary points w_{12} and w_{34}. More precisely, to the quarks v_1 and v_2 are associated the Melznak set $s = \{w_{12}, t_{12}\}$ and to the antiquarks v_3 and v_4 are associated $\bar{s} = \{w_{34}, t_{34}\}$, so that the triangles (w_{12}, v_1, v_2) and (w_{34}, v_3, v_4) are equilateral. The length of the minimal Steiner tree linking v_1, v_2 to v_3, v_4, which involves the optimisation of several continuous variables is equal to the maximal distance from s to $\bar{s}$, which is just a discrete optimisation ,and thus very much faster.

auxiliary pieces of the picture. So experts on multiquark physics can seek positions at Hollywood!

A simplified version of the connected interaction V_s can be found in the work by Barbour and Ponting [267,268]. A set of Jacobi coordinates is shown in Fig. 23. It reads (besides the centre of mass)

$$\boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1 \,, \quad \boldsymbol{y} = \boldsymbol{r}_4 - \boldsymbol{r}_3 \,, \quad \boldsymbol{z} = \frac{\boldsymbol{r}_4 + \boldsymbol{r}_3 - \boldsymbol{r}_2 - \boldsymbol{r}_1}{\sqrt{2}} \,, \tag{74}$$

so that the intrinsic part of the four-body Hamiltonian reads

$$H_4 = \frac{\boldsymbol{p}_x^2}{m} + \frac{\boldsymbol{p}_y^2}{m} + \frac{\boldsymbol{p}_z^2}{m} + V \,. \tag{75}$$

In the case of a T-baryonium, with colour $\bar{3}$ coupling of the two quarks, and 3 for the antiquarks, Barbour and Ponting adopted a confining interaction

$$V = a(x + y + z/\sqrt{2}) \,. \tag{76}$$

Then H_4 exactly splits into three independent pieces, and the ground state is at $e_0\, a^{2/3}\, m^{-1/3}[2 + 2^{-1/3}]$ above the threshold at $2\, e_0\, a^{2/3}\, m^{-1/3}$. Here e_0 is the negative of the first zero of the Airy function, see Sec. 3.3.

Now, if one gives up the square geometry of Barbour and Ponting, and minimises the string potential by varying the location of the junctions, one gets a lower energy, but still above the threshold, for equal masses.

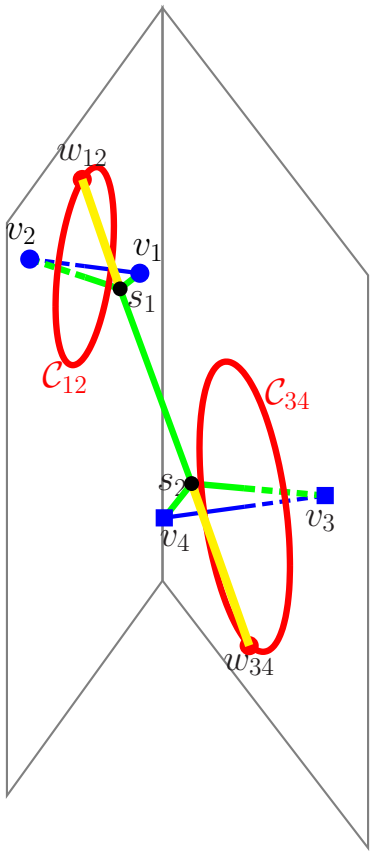

Figure 22: The minimal length of the Steiner tree $v_1s_1 + v_2s_1 + s_1s_2 + s_2v_3 + s_2v_4$ linking the quarks to the antiquarks is the maximal distance $w_{12}w_{34}$ between their Melznak's circles. Any point of the circle $\mathcal{C}_{12}$ makes an equilateral triangle with the quarks v_1 and v_2. The same holds in the antiquark sector.

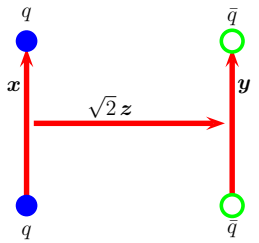

Figure 23: Confining interaction for a tetraquark in the model by Barbour and Ponting : $V = a\,(x + y + z)$

5.5.4 Rigorous results

We now discuss the four-body energy with the full linear potential V_4 of Eq. (73), starting with the exact result, and then, in the next subsection the numerical results.

There are not many exact results for the four-body problem, and with the cumbersome potential (73), it looks difficult to derive a simple upper bound. In particular, V_4 is smaller than the flip-flop V_f alone, which in turn is smaller than the potential $a\,r_{13}+a\,r_{24}$ which governs the threshold. This means that the effective meson–meson interaction is attractive. In a world with on or two dimensions, this would imply binding. But in our valley of tears in three dimensions, this is not the case.

Nevertheless, it is easily found from the construction of Fig. 22 that

$$V_s \leq a\left[(x+y)\frac{\sqrt{3}}{2}+z\sqrt{2}\right]\ , \tag{77}$$

and it has been shown [265] that this bound also holds for V_4. Thus one gets a upper bound

$$H_4 \leq \frac{\boldsymbol{p}_x^2}{m}+\frac{\boldsymbol{p}_y^2}{m}+\frac{\boldsymbol{p}_z^2}{m}+a\left[(x+y)\frac{\sqrt{3}}{2}+z\sqrt{2}\right]\ , \tag{78}$$

which is exactly a sum of three independent terms. Is is easily found that in this model $(QQ\bar{q}\bar{q})$ becomes stable for a mass ratio $M/m \gtrsim 6402$. This is of course very crude, as the inequality in the potential is saturated only when the circles in Fig. 22 are in the same plane, and do not overlap.

5.5.5 Numerical results

The four-body problem with the interaction V_4 can be solved using standard techniques. Here, the difficult y comes from the numerical estimate of the matrix elements of the potential. It is found that binding is obtained even for equal masses [264], and, of course, it becomes better for $(QQ\bar{q}\bar{q})$. However, the minimisation in (73) and in the flip-flop itself implies a continuous rotation of the internal colour degree of freedom. Hence colour cannot contribute to the antisymmetrisation. In other words, the stability if for distinguishable quarks and antiquarks. The connected term, or Steiner tree, contributes marginally, as compared to the flip-flop.

5.5.6 String model of pentaquark

The model can be adapted to the case of pentaquarks, i.e., four quarks and one antiquark [269]. The potential is pictured in Fig. 24. Again, the interaction contains flip-flop terms, i.e., the lowest of the meson + baryon string potential, and some connected diagrams, the rule being to take the minimum.

Again, the calculation corresponds to distinguishable quarks, even in the case where for simplicity the same mass is adopted. Then it is found that the pentaquark is stable.

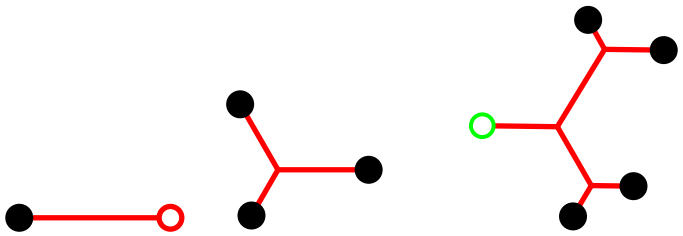

Figure 24: Simple string model for the pentaquark configurations

5.5.7 String model of hexaquarks

In a recent study, six-quark configurations were studied, both $(q^3\bar{q}^3)$ and (q^6) [270].

The string potential for $(q^3\bar{q}^3)$ is shown in Fig. 25. Missing is a term consisting of a meson and a separated tetraquark. For quarks and antiquarks with equal masses, but distinguishable, a weak binding is obtained. The lowest threshold is $(q\bar{q})^3$. If one introduces two different masses and look at $(Q^3\bar{q}^3)$, the binding first is improved when M/m increases from $M/m = 1$. But for very large M/m, the lowest threshold becomes $(QQQ) + (\bar{q}\bar{q}\bar{q})$ and $(Q^3\bar{q}^3)$ becomes unstable.

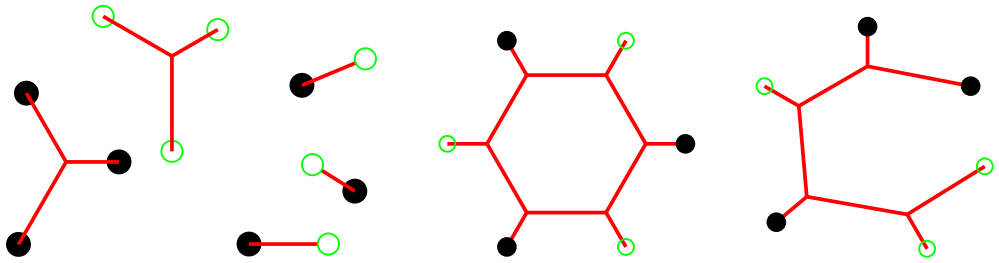

Figure 25: Potential for $(q^3\bar{q}^3)$: disconnected terms of baryon–antibaryon type of three-meson type (with suitable minimisation over the permutations) and some connected terms.

In the case of (q^6) the potential is either of baryon–baryon type, with all possible permutations, or given by a connected string. See Fig. 26. For equal masses, a comfortable binding is obtained. For two different masses, i.e., (Q^3q^3), the binding deteriorates. As the mass ratio M/m further increases, the 6-quark system becomes unbound and the ground state consists of two isolated baryons, $(QQQ) + (qqq)$.

5.5.8 The next challenge: combining string and full antisymmetrisation

The above string model, with mainly a flip-flop interaction and some connected contributions, has many appealing features. However, it is not suited for implementing the constraints of antisymmetrisation, and it tends to produce several stable multiquarks

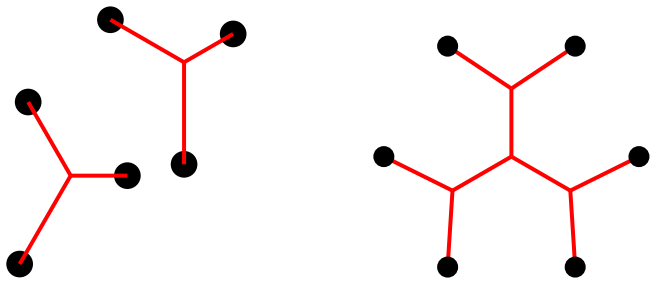

Figure 26: Contributions to the linear part of the (q^6) potential, disconnected and conected terms. A minimisation over the permutations is implied.

that are not physical. Consider for instance the tetraquark case. The potential is taken as the minimum (in units of the string tension) of $r_{13} + r_{24}$ which corresponds to a singlet–singlet state of colour, or $r_{14} + r_{23}$ which corresponds to another singlet–singlet, and the double-Y string which corresponds to $(\bar{3} - 3)_1$. For distinguishable quarks and anti-quarks, this is plausible, and the string potential is just a Born–Oppenheimer potential, once the coloured gluon field is minimised. Some change is required to implement this dynamics with full account of the symmetry constraints.

5.6 The light pentaquark

The claim by the LEPS collaboration [271] that there exists a rather narrow baryon θ^+ with mass about 1.54 GeV, charge $Q = +1$ and strangeness $S = +1$ stimulated much activity in the physics of hadron.

First, other experimental groups looked back at their old data already on tapes, and also found some evidence for this state or for its SU(3)$_F$ partners (In the paper by Diakonov et al. [272] having motivated the experimental search, the θ^+ supposedly belongs to an antidecuplet $\bar{10}$). Later, the experimental evidence became much weaker. For a summary, see, e.g., the review by PDG [].

Second, the original calculation within chiral dynamics was revisited [273, 274] and led to different conclusions. The pentaquark was also studied with QCD sum rules or Lattice QCD, with a variety of conclusions. There were also studies within potential models, with detailed five-body calculation. See, for instance, [275]. The light pentaquark can not be reproduced in such calculations, unless one introduces ad-hoc extra assumptions.

6 Outlook

Important problems never receive a definite solution
Confucius

About 50 years after the first speculations on quarks, what does remain? Let us just list of few items for the discussions after the lectures.

$SU(3)_F$ symmetry is now becoming somewhat controversial. Sometimes, the $SU(3)_F$ symmetry is almost exact. This is the case for the rates of $J/\psi \to p\bar{p},\ \Lambda\bar{\Lambda}, \ldots$. In other cases, $SU(3)_F$ looks badly broken. For instance, if one looks at K to π production in high-energy collisions, one finds a large "strangeness suppression". Perhaps the elementary couplings for $u\bar{u}$ or $s\bar{s}$ production are very similar, but the small differences in masses are amplified by a tunnelling factor which is exponential, like in the Gamow theory of α decay.

From flavour symmetry to flavour independence In hadron spectroscopy, it is delicate to attempt a blind extension of $SU(3)_F$ to SU(4) or SU(5) to include charm and beauty. The breaking is too large to be treated as a perturbation. A new understanding of the symmetry has been provided by the quark model. The basic interaction is flavour independent, or has a well-defined flavour dependence in the spin-spin and spin-orbit terms. Then the same basic interaction is used to build the various $(q\bar{q}')$ mesons and $(qq'q'')$ baryons.

The bootstrap idea was conceptually superb, but operationally somewhat inefficient. Still, a kind of duality exists[24]: for instance, it is natural to consider the Δ baryon in some context the (qqq) partner of $N = (qqq)$ with all quark spins aligned, but to describe the pion-nucleus scattering, there is nothing better than Δ as a $\pi - N$ resonance. When the quark model was invented, it was considered as a relief, with at last a systematic and coherent scheme to describe the hadrons. Nowadays, for any hadron a of any mass m, one finds a pair of existing hadrons b and c and an effective Lagrangian coupling a, b and c, such that a appears as a kind of molecule. The overall picture is seemingly lost. Is that a big step backwards?

From quark models to QCD This will be covered in some of the next lectures. It is fascinating how a somewhat empirical model, but based on the acute observation of facts, became a beautiful theory.

A premium to simplicity The quark model, even in its simplest variant, works rather well. Presumably, the quark model incorporates a lot of subtle dynamical effects in an effective way, and enables one to make successful predictions.

[24] G.F. Chew, private communication, years ago, that is very thankfully remembered

Hadron dynamics beyond hadron decay An ambitious application of the quark model deals with the hadron–hadron interaction. This is the continuum part of the sections devoted to loop corrections and to multiquark spectroscopy.

Let us mention the calculation of the KN interaction in the exotic $S = +1$ sector of strangeness, and the extensive work done to understand the short-range part of the nucleon–nucleon interaction in terms of quarks.

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